

## **The Rational valued characters of the Group ( $Q_{2m} \times C_5$ ) When m is prime number**

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### **1- Abstract**

The main purpose of this paper is to find the rational valued characters table of the group (  $Q_{2m} \times C_5$  ), When m is prime number , where  $Q_{2m}$  is denoted to Quaternion group , and  $C_5$  is the cyclic group of order 5 .

In this paper .we find the general form of t the rational valued characters table of the group (  $Q_{2m} \times C_5$  ) is the tensor product of the rational valued characters of  $Q_{2m}$  and  $C_5$

### **المستخلص**

الهدف الرئيسي لهذا البحث هو ايجاد جدول الشواخص ذات القيم النسبيه للزمرة  $Q_{2m} \times C_5$ , عندما m عدد اولي ,حيث يرمز للزمرة الرباعيه بالرمز  $Q_{2m}$ , ويرمز للزمرة الدائريه ذات الرتبه 5 بالرمز  $C_5$  .  
في هذا البحث وجدنا جدول الشواخص ذات القيم النسبيه للزمرة  $Q_{2m} \times C_5$  باستخدام ضرب tensor للشواخص ذات القيم النسبيه ل  $Q_{2m}$  و  $C_5$  .

### **2- Introduction**

Let G be a finite group ,two elements of G are said to be  $\Gamma$  –conjugate if the cyclic subgroups they generate are equivalence relation on G . Its classes are called  $\Gamma$  –classes the Z –valued class function on the group G , which is constant on the  $\Gamma$  –classes forms a finitely generated abelian group  $cf(G, Z)$  of rank equal to the number of  $\Gamma$  –classes .

The intersection of  $cf(G, Z)$  with the group of all generalized characters of G ,  $R(G)$  is a normal subgroup of  $cf(G, Z)$  dented by  $R(G)$  Each element in  $R(G)$  can be written as

$u_1 \theta_1 + u_2 \theta_2 + \dots + u_l \theta_l$ , where l is the number of  $\Gamma$  – classes ,  $u_1, u_2, \dots, u_l \in Z$  and  $\theta_i = \sum_{\sigma \in Gal(Q(\lambda_i)/Q)} \sigma(\lambda_i)$

Where  $\lambda_i$  is an irreducible character of the group G and  $\sigma$  is any element in Galios group  $Gal(Q(\lambda_i)/Q)$  .

Let  $\equiv^*(G)$  denotes the  $l \times l$  matrix which the rows corresponds to the  $\theta_i$ 's and columns corresponds to the  $\Gamma$  classes of G .

In 1995 N. R. Mahamood [ 3 ] studied the factor group  $cf(Q_{2m}, Z) / R(Q_{2m})$  . The aim of this paper is to find  $\equiv^*(Q_{2m} \times C_5)$  and determine general (  $m + 3 \times m + 3$  ) matrix form of the rational valued characters table of the group (  $Q_{2m} \times C_5$  ) .

### **3- Preliminaries**

#### **Definition ( 3.1 ) [ 3 ]**

For each positive integer m , the generalized Quaternion Group  $Q_{2m}$  of order 4 m with two generators x and y satisfies

$$Q_{2m} = \{ x^h y^k, 0 \leq h \leq 2m - 1, k = 0,1 \}$$

Which has the following properties

$$\{ x^{2m} = y^4 = 1, y \times^m y^{-1} = x^{-m} \}$$

**Definition ( 3.2) [ 3 ]**

**Table ( 1 )**

	$X^k$	$X^k y$
$\psi_1$	1	1
$\psi_2$	1	-1
$\psi_3$	$(-1)^k$	$i(-1)^k$
$\psi_4$	$(-1)^k$	$i(-1)^{k+1}$

There are two types of irreducible characters one of them is the character of the linear representations  $R_1, R_2, R_3$  and  $R_4$  which are denoted by  $\psi_1, \psi_2, \psi_3$  and  $\psi_4$  . respectively as in the table(1)

where  $0 \leq k \leq 2m - 1$

The rest characters of irreducible representations  $T_h$  of degree 2 are

denoted by  $\lambda_h$  such that :

$$\lambda_h ( X^k ) = \omega^{hk} + \omega^{-hk} = e^{\pi i h k / m} + e^{-\pi i h k / m} = 2 \cos ( \pi h k / m )$$

we are denoted to  $\omega^{hk} + \omega^{-hk}$  by  $V_{hk}$  , thus

$$V_{hk} = V_{2m-hk} , V_m = -2 , V_{2m} = 2$$

also we will write  $V_{J(hk)}$  such that

$J ( hk ) = \min \{ hk ( \text{mod } 2m ) , 2m - hk ( \text{mod } 2m ) \}$  , in the character table of the quaternion group  $Q_{2m}$  when  $m$  is prime number , where  $V_{J(hk)} = 2 \cos ( \pi J ( hk ) / m )$  ,  $\lambda_h ( X^k y ) = 0$

Where  $0 \leq k \leq 2m - 1$  ,  $1 \leq h \leq m - 1$  and  $\omega = e^{2\pi i / 2m}$

So there are  $m + 3$  irreducible characters of  $Q_{2m}$  .

**Theorem ( 3.3) [ 1 ]**

Let  $T_1 : G_1 \longrightarrow GL ( n , k )$  and  $T_2 : G_2 \longrightarrow GL ( m , k )$  are two irreducible representation of the group  $G_1$  and  $G_2$  with characters  $\lambda_1$  and  $\lambda_2$  respectively , then  $T_1 \otimes T_2$  is irreducible representation of the group  $G_1 \times G_2$  with character  $\lambda_1 \cdot \lambda_2$  .

**Proposition ( 3.4) [ 2 ]**

$$\text{The rational valued characters } \theta_i = \sum_{\sigma \in \text{Gal} ( Q ( \lambda_i ) / Q )} \sigma ( \lambda_i )$$

Form basis for  $R ( \bar{G} )$  , where  $\lambda_i$  are the irreducible characters of  $G$  and their numbers are equal to the number of all distinct  $\Gamma$ - classes of  $G$  .

**The rational character table of the quaternion group  $Q_{2m}$  when  $m$  is prime number( 3.5 ) [ 3 ]**

The rational characters table of  $Q_{2m}$  when  $m$  is prime number is given in the following table :  
 $\equiv^* ( Q_{2m} ) =$

**Table(2)**

$\Gamma$ -classes	[1]	$[x^2]$	$[x^m]$	[x]	[y]
$\theta_1$	1	1	1	1	1
$\theta_2$	m-1	-1	m-1	-1	0
$\theta_3$	1	1	1	1	-1
$\theta_4$	m-1	-1	-(m-1)	1	0
$\theta_5$	2	2	-2	-2	0

**The Group  $Q_{2m} \times C_5$  (3.6)**

The direct product group  $Q_{2m} \times C_5$ , where  $C_5$  is a cyclic group of order 5 then  $|Q_{2m} \times C_5| = 20m$  since, the irreducible representation of the group  $Q_{2m} \times C_5$  are the tensor products of those of  $Q_{2m}$  and those of  $C_5$ . The group  $C_5$  has five irreducible representation, their characters  $\lambda_1, \lambda_2, \lambda_3, \lambda_4$  and  $\lambda_5$  are given in the table (2):

Table (3)

CLa	I	r	r <sup>2</sup>	r <sup>3</sup>	r <sup>4</sup>
CLa	1	1	1	1	1
$\lambda_1$	1	1	1	1	1
$\lambda_2$	1	$\epsilon$	$\epsilon^2$	$\epsilon^3$	$\epsilon^4$
$\lambda_3$	1	$\epsilon^2$	$\epsilon^4$	$\epsilon$	$\epsilon^3$
$\lambda_4$	1	$\epsilon^3$	$\epsilon$	$\epsilon^4$	$\epsilon^2$
$\lambda_5$	1	$\epsilon^4$	$\epsilon^3$	$\epsilon^2$	$\epsilon$

According to theorem (3.3), each irreducible characters  $\lambda_i$  of  $Q_{2m}$  defines five irreducible characters  $\lambda_{i1}, \lambda_{i2}, \lambda_{i3}, \lambda_{i4}$  and  $\lambda_{i5}$  such that  $\lambda_{i1} = \lambda_i \lambda_1, \lambda_{i2} = \lambda_i \lambda_2, \lambda_{i3} = \lambda_i \lambda_3,$

$$\lambda_{i4} = \lambda_i \lambda_4 \text{ and } \lambda_{i5} = \lambda_i \lambda_5 \text{ of } Q_{2m} \times C_5 \text{ then}$$

$$\equiv (Q_{2m} \times C_5) = \equiv (Q_{2m}) \otimes \equiv (C_5)$$

**Example (3.7)**

To find characters table  $Q_{14} \times C_5$  from (3.3) we have the characters table of  $\equiv (Q_{14})$

cl $\alpha$	[I]	[x <sup>2</sup> ]	[x <sup>4</sup> ]	[x <sup>6</sup> ]	[x <sup>7</sup> ]	[x]	[x <sup>3</sup> ]	[x <sup>5</sup> ]	[y]	[xy]
cl $\alpha$	1	2	2	2	1	2	2	2	7	7
$\psi_1$	1	1	1	1	1	1	1	1	1	1
$\lambda_2$	2	V <sub>4</sub>	V <sub>6</sub>	V <sub>2</sub>	2	V <sub>2</sub>	V <sub>6</sub>	V <sub>4</sub>	0	0
$\lambda_4$	2	V <sub>6</sub>	V <sub>2</sub>	V <sub>4</sub>	2	V <sub>4</sub>	V <sub>2</sub>	V <sub>6</sub>	0	0
$\lambda_6$	2	V <sub>2</sub>	V <sub>4</sub>	V <sub>6</sub>	2	V <sub>6</sub>	V <sub>4</sub>	V <sub>2</sub>	0	0
$\psi_2$	1	1	1	1	1	1	1	1	-1	-1
$\lambda_1$	2	V <sub>2</sub>	V <sub>4</sub>	V <sub>6</sub>	-2	V <sub>1</sub>	V <sub>3</sub>	V <sub>5</sub>	0	0
$\lambda_3$	2	V <sub>6</sub>	V <sub>2</sub>	V <sub>4</sub>	-2	V <sub>3</sub>	V <sub>5</sub>	V <sub>1</sub>	0	0
$\lambda_5$	2	V <sub>4</sub>	V <sub>6</sub>	V <sub>2</sub>	-2	V <sub>5</sub>	V <sub>1</sub>	V <sub>3</sub>	0	0
$\psi_3$	1	1	1	1	-1	-1	-1	-1	i	-i
$\psi_4$	1	1	1	1	-1	-1	-1	-1	-i	i

Table (4)

, where  $V_i = 2 \cos(\pi_i / 7)$ .

$$V_{2m} = 2, V_m = -2.$$

By Theorem (3.3) the characters table of  $Q_{14} \times C_5$  can be written as follows:

$$\equiv (Q_{14} \times C_5) = \equiv (Q_{14}) \otimes \equiv (C_5),$$

Then  $\equiv (Q_{14} \times C_5)$  is given in the table (14)

**4 . The main results**

**Example ( 4.1 )**

To construct the rational valued characters table of  $(Q_{2m} \times C_5)$  when have to do the following from table(14) we have the characters table of  $(Q_{14} \times C_5)$  By the definition of  $(Q_{2m} \times C_5)$  :then,

$$\equiv (Q_{14} \times C_5) \equiv (Q_{14}) \otimes \equiv (C_5)$$

To calculate the rational valued characters table of  $(Q_{14} \times C_5)$

$$\theta_{11} = \psi_{11}, \theta_{12} = \psi_{12} + \psi_{13} + \psi_{14} + \psi_{15}$$

$$\theta_{31} = \psi_{21}, \theta_{32} = \psi_{22} + \psi_{23} + \psi_{24} + \psi_{25}$$

$$\theta_{51} = \psi_{31} + \psi_{41}, \theta_{52} = \psi_{32} + \psi_{33} + \psi_{34} + \psi_{35} + \psi_{42} + \psi_{43} + \psi_{44} + \psi_{45}$$

The elements of  $\text{Gal}(\lambda_{1i})/Q$  are :

$$\{ \sigma_{1i}, \sigma_{3i}, \sigma_{5i} \}$$

$$\sigma_{1i}(\lambda_{1i}) = \lambda_{1i}, \sigma_{3i}(\lambda_{1i}) = \lambda_{3i}, \sigma_{5i}(\lambda_{1i}) = \lambda_{5i}$$

where  $i = 1, 2, 3, 4, 5$

By proposition (3.4)

1- (I) if  $i = 1$

$$\theta_{41} = \sigma_{11}(\lambda_{11}) + \sigma_{31}(\lambda_{11}) + \sigma_{51}(\lambda_{11}) = \lambda_{11} + \lambda_{31} + \lambda_{51}$$

$$\theta_{41}([I, I]) = 2 + 2 + 2 = 6$$

$$\theta_{41}([I, r]) = 2 + 2 + 2 = 6$$

$$\theta_{41}([x^2, I]) = V_2 + V_6 + V_4 = -1$$

$$\theta_{41}([x^2, r]) = V_2 + V_6 + V_4 = -1$$

$$\theta_{41}([x^7, I]) = (-2) + (-2) + (-2) = -6$$

$$\theta_{41}([x^7, r]) = (-2) + (-2) + (-2) = -6$$

$$\theta_{41}([x, I]) = V_1 + V_3 + V_5 = 1$$

$$\theta_{41}([x, r]) = V_1 + V_3 + V_5 = 1$$

$$\theta_{41}([y, I]) = 0 + 0 + 0 = 0$$

$$\theta_{41}([y, r]) = 0 + 0 + 0 = 0$$

1- (II) if  $i = 2, 3, 4, 5$

$$\theta_{42} = \sigma_{12}(\lambda_{12}) + \sigma_{13}(\lambda_{13}) + \sigma_{14}(\lambda_{14}) + \sigma_{15}(\lambda_{15}) + \sigma_{32}(\lambda_{12}) + \sigma_{33}(\lambda_{13}) + \sigma_{34}(\lambda_{14}) + \sigma_{35}(\lambda_{15}) + \sigma_{52}(\lambda_{12}) + \sigma_{53}(\lambda_{13}) + \sigma_{54}(\lambda_{14}) + \sigma_{55}(\lambda_{15})$$

$$\theta_{42}([I, I]) = 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 24$$

$$\theta_{42}([I, r]) = 2\epsilon + 2\epsilon^2 + 2\epsilon^3 + 2\epsilon^4 + 2\epsilon + 2\epsilon^2 + 2\epsilon^3 + 2\epsilon^4 + 2\epsilon + 2\epsilon^2 + 2\epsilon^3 + 2\epsilon^4$$

$$= 2(\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4) + 2(\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4) + 2(\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4)$$

$$= 2(-1) + 2(-1) + 2(-1) = -6$$

$$\theta_{42}([x^2, I]) = V_2 + V_2 + V_2 + V_2 + V_6 + V_6 + V_6 + V_6 + V_4 + V_4 + V_4 + V_4$$

$$= (V_2 + V_6 + V_4) + (V_2 + V_6 + V_4) + (V_2 + V_6 + V_4) + (V_2 + V_6 + V_4)$$

$$= -1 - 1 - 1 - 1 = -4$$

$$\theta_{42}([x^2, r]) = \epsilon V_2 + \epsilon^2 V_2 + \epsilon^3 V_2 + \epsilon^4 V_2 + \epsilon V_6 + \epsilon^2 V_6 + \epsilon^3 V_6 + \epsilon^4 V_6 + \epsilon V_4 + \epsilon^2 V_4 + \epsilon^3 V_4 + \epsilon^4 V_4$$

$$= V_2(\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4) + V_6(\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4) + V_4(\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4)$$

$$= -V_2 - V_6 - V_4 = -(V_2 + V_6 + V_4) = -(-1) = 1$$

$$\theta_{42}([x^7, I]) = (-2) + (-2) + (-2) + (-2) + (-2) + (-2) + (-2) + (-2) + (-2) + (-2) + (-2) + (-2)$$

$$= -24$$

$$\theta_{42}([x^7, r]) = (-2\epsilon) + (-2\epsilon^2) + (-2\epsilon^3) + (-2\epsilon^4) + (-2\epsilon) + (-2\epsilon^2) + (-2\epsilon^3) + (-2\epsilon^4) + (-2\epsilon) + (-2\epsilon^2) + (-2\epsilon^3) + (-2\epsilon^4)$$

$$= (-2)(-1) + (-2)(-1) + (-2)(-1) = 6$$

$$\theta_{42}([x, I]) = V_1 + V_1 + V_1 + V_1 + V_3 + V_3 + V_3 + V_3 + V_5 + V_5 + V_5 + V_5$$

$$= (V_1 + V_3 + V_5) + (V_1 + V_3 + V_5) + (V_1 + V_3 + V_5) + (V_1 + V_3 + V_5)$$

$$= 1 + 1 + 1 + 1 = 4$$

$$\begin{aligned} \theta_{42}([x, r]) &= \epsilon V_1 + \epsilon^2 V_1 + \epsilon^3 V_1 + \epsilon^4 V_1 + \epsilon V_3 + \epsilon^2 V_3 + \epsilon^3 V_3 + \epsilon^4 V_3 \\ &+ \epsilon V_5 + \epsilon^2 V_5 + \epsilon^3 V_5 + \epsilon^4 V_5 = (-1) V_1 + (-1) V_3 + (-1) V_5 \\ &= -1 (V_1 + V_3 + V_5) = (-1) (1) = -1 \end{aligned}$$

$$\theta_{42}([y, I]) = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0$$

$$\theta_{42}([y, r]) = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0$$

Also  $\sigma_{1i}(\lambda_{2i}) = \lambda_{2i}$ ,  $\sigma_{3i}(\lambda_{2i}) = \lambda_{4i}$ ,  $\sigma_{5i}(\lambda_{2i}) = \lambda_{6i}$  where  $i = 1, 2, 3, 4, 5$

**By proposition (3.4)**

1- (I) if  $i = 1$

$$\theta_{21} = \sigma_{11}(\lambda_{21}) + \sigma_{31}(\lambda_{21}) + \sigma_{51}(\lambda_{21}) = \lambda_{21} + \lambda_{41} + \lambda_{61}$$

$$\theta_{21}([I, I]) = 2 + 2 + 2 = 6$$

$$\theta_{21}([I, r]) = 2 + 2 + 2 = 6$$

$$\theta_{21}([x^2, I]) = V_4 + V_6 + V_2 = -1$$

$$\theta_{21}([x^2, r]) = V_4 + V_6 + V_2 = -1$$

$$\theta_{21}([x^7, I]) = 2 + 2 + 2 = 6$$

$$\theta_{21}([x^7, r]) = 2 + 2 + 2 = 6$$

$$\theta_{21}([x, I]) = V_2 + V_4 + V_6 = -1$$

$$\theta_{21}([x, r]) = V_2 + V_4 + V_6 = -1$$

$$\theta_{21}([y, I]) = 0 + 0 + 0 = 0$$

$$\theta_{21}([y, r]) = 0 + 0 + 0 = 0$$

(II) if  $i = 2, 3, 4, 5$

$$\begin{aligned} \theta_{22} &= \sigma_{12}(\lambda_{22}) + \sigma_{13}(\lambda_{23}) + \sigma_{14}(\lambda_{24}) + \sigma_{15}(\lambda_{25}) + \sigma_{32}(\lambda_{22}) + \sigma_{33}(\lambda_{23}) \\ &+ \sigma_{34}(\lambda_{24}) + \sigma_{35}(\lambda_{25}) + \sigma_{52}(\lambda_{22}) + \sigma_{53}(\lambda_{23}) + \sigma_{54}(\lambda_{24}) + \sigma_{55}(\lambda_{25}) \\ &= \lambda_{22} + \lambda_{23} + \lambda_{24} + \lambda_{25} + \lambda_{42} + \lambda_{43} + \lambda_{44} + \lambda_{45} + \lambda_{62} + \lambda_{63} + \lambda_{64} + \lambda_{65} \end{aligned}$$

$$\theta_{22}([I, I]) = 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 24$$

$$\begin{aligned} \theta_{22}([I, r]) &= 2\epsilon + 2\epsilon^2 + 2\epsilon^3 + 2\epsilon^4 + 2\epsilon + 2\epsilon^2 + 2\epsilon^3 + 2\epsilon^4 + \\ &2\epsilon + 2\epsilon^2 + 2\epsilon^3 + 2\epsilon^4 = 2(-1) + 2(-1) + 2(-1) = -6 \end{aligned}$$

$$\begin{aligned} \theta_{22}([x^2, I]) &= (V_4 + V_4 + V_4 + V_4) + (V_6 + V_6 + V_6 + V_6) + (V_2 + V_2 + V_2 + V_2) = (-1) \\ &+ (-1) + (-1) + (-1) = -4 \end{aligned}$$

$$\begin{aligned} \theta_{22}([x^2, r]) &= \epsilon V_4 + \epsilon^2 V_4 + \epsilon^3 V_4 + \epsilon^4 V_4 + \epsilon V_6 + \epsilon^2 V_6 + \epsilon^3 V_6 + \epsilon^4 V_6 \\ &+ \epsilon V_2 + \epsilon^2 V_2 + \epsilon^3 V_2 + \epsilon^4 V_2 = V_4(-1) + V_6(-1) + V_2(-1) \\ &= -(V_4 + V_6 + V_2) = -1(-1) = 1 \end{aligned}$$

$$\theta_{22}([x^7, I]) = 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 24$$

$$\begin{aligned} \theta_{22}([x^7, r]) &= 2\epsilon + 2\epsilon^2 + 2\epsilon^3 + 2\epsilon^4 + 2\epsilon + 2\epsilon^2 + 2\epsilon^3 + 2\epsilon^4 + 2\epsilon + \\ &2\epsilon^2 + 2\epsilon^3 + 2\epsilon^4 = 2(-1) + 2(-1) + 2(-1) = -6 \end{aligned}$$

$$\begin{aligned} \theta_{22}([x, I]) &= V_2 + V_2 + V_2 + V_2 + V_4 + V_4 + V_4 + V_4 + V_6 + V_6 + V_6 + V_6 \\ &= (-1) + (-1) + (-1) + (-1) = -4 \end{aligned}$$

$$\begin{aligned} \theta_{22}([x, r]) &= \epsilon V_2 + \epsilon^2 V_2 + \epsilon^3 V_2 + \epsilon^4 V_2 + \epsilon V_4 + \epsilon^2 V_4 + \epsilon^3 V_4 + \epsilon^4 V_4 \\ &+ \epsilon V_6 + \epsilon^2 V_6 + \epsilon^3 V_6 + \epsilon^4 V_6 = V_2(-1) + V_4(-1) + V_6(-1) \\ &= -(V_2 + V_4 + V_6) = -1(-1) = 1 \end{aligned}$$

$$\theta_{22}([y, I]) = 0, \theta_{22}([y, r]) = 0$$

The elements  $[I, r], [I, r^2], [I, r^3], [I, r^4]$  are in the same  $\Gamma$ -conjugate and  $[x^2, I], [x^4, I], [x^6, I]$  are in the same  $\Gamma$ -conjugate and  $[x^2, r], [x^2, r^2], [x^2, r^3], [x^2, r^4], [x^4, r], [x^4, r^2], [x^4, r^3], [x^4, r^4], [x^6, r], [x^6, r^2], [x^6, r^3], [x^6, r^4]$  are in the same  $\Gamma$ -conjugate and  $[x^7, r], [x^7, r^2], [x^7, r^3], [x^7, r^4]$

are in the same  $\Gamma$ -conjugate and  $[x, I], [x^3, I], [x^5, I]$  are in the same  $\Gamma$ -conjugate and  $[x, r], [x, r^2], [x, r^3], [x, r^4], [x^3, r], [x^3, r^2], [x^3, r^3], [x^3, r^4], [x^5, r], [x^5, r^2], [x^5, r^3], [x^5, r^4]$  are in the same  $\Gamma$ -conjugate and  $[y, I], [xy, I]$  are in the same  $\Gamma$ -conjugate and  $[y, r], [y, r^2], [y, r^3], [y, r^4], [xy, r], [xy, r^2], [xy, r^3], [xy, r^4]$  are in the same  $\Gamma$ -conjugate.

$\cong^* (Q_{14} \times C_5)$

$cl_\alpha$	[I, I]	[I, r]	[x <sup>2</sup> , I]	[x <sup>2</sup> , r]	[x <sup>7</sup> , I]	[x <sup>7</sup> , r]	[x, I]	[x, r]	[y, I]	[y, r]
$ cl_\alpha $	1	1	2	2	1	1	2	2	7	7
$\theta_{11}$	1	1	1	1	1	1	1	1	1	1
$\theta_{12}$	4	-1	4	-1	4	-1	4	-1	4	-1
$\theta_{21}$	6	6	-1	-1	6	6	-1	-1	0	0
$\theta_{22}$	24	-6	-4	1	24	-6	-4	1	0	0
$\theta_{31}$	1	1	1	1	1	1	1	1	-1	-1
$\theta_{32}$	4	-1	4	-1	4	-1	4	-1	-4	1
$\theta_{41}$	6	6	-1	-1	-6	-6	1	1	0	0
$\theta_{42}$	24	-6	-4	1	-24	6	4	-1	0	0
$\theta_{51}$	2	2	2	2	-2	-2	-2	-2	0	0
$\theta_{52}$	8	-2	8	-2	-8	2	-8	2	0	0

Table (5)

**Theorem (4.2)**

The rational valued characters table of the group  $(Q_{2m} \times C_5)$  when m is prime number is given as follows :

$\cong^* (Q_{2m} \times C_5) = \cong^* (Q_{2m}) \otimes \cong^* (C_5)$

Proof : Since  $C_5 = \{ I, r, r^2, r^3, r^4 \}$

since

$cl_\alpha$	I	R	r <sup>2</sup>	r <sup>3</sup>	r <sup>4</sup>
$\lambda_1$	1	1	1	1	1
$\lambda_2$	1	$\epsilon$	$\epsilon^2$	$\epsilon^3$	$\epsilon^4$
$\lambda_3$	1	$\epsilon^2$	$\epsilon^4$	$\epsilon$	$\epsilon^3$
$\lambda_4$	1	$\epsilon^3$	$\epsilon$	$\epsilon^4$	$\epsilon^2$
$\lambda_5$	1	$\epsilon^4$	$\epsilon^3$	$\epsilon^2$	$\epsilon$

Table(6)

and  $\cong^* (C_5) =$

$\Gamma$ -classes	I	r
$\theta_1$	1	1
$\theta_2$	4	-1

Table(7)

Then,  $\lambda_1(I) = \theta_1(I) = 1$

$\lambda_1(r) = \lambda_1(r^2) = \lambda_1(r^3) = \lambda_1(r^4) = \theta_1(r^i) = 1, i = 1, \dots, 4$

$\lambda_2(I) + \lambda_3(I) + \lambda_4(I) + \lambda_5(I) = \theta_2(I) = 1 + 1 + 1 + 1 = 4$

$\lambda_2(r) + \lambda_3(r) + \lambda_4(r) + \lambda_5(r) = \theta_2(r) = \epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4 = -1$

Where  $\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4 = -1$

From the definition of  $(Q_{2m} \times C_5)$  theorem (3.3)

$\cong^* (Q_{2m} \times C_5) = \cong^* (Q_{2m}) \otimes \cong^* (C_5)$

each element in  $Q_{2m} \times C_5$

Let  $u \in Q_{2m} \times C_5$  then  $u = (q, c)$  when

$q \in Q_{2m}$  and  $c \in C_5, C = r^i, i = 0, \dots, 4$

$q = x^s y^k, 0 \leq s \leq 2m, k = 0, 1$

and each irreducible character of  $Q_{2m} \times C_5$  is

$$\lambda_{ij} = \lambda_i \lambda_j$$

where  $\lambda_i$  is an irreducible character of  $Q_{2m}$

and  $\lambda_j$  is an irreducible character of  $C_5$ , then

$$\lambda_{ij}(u) = \lambda_{ij}(q, c) = \lambda_i(q) \cdot \lambda_j(c) = \begin{cases} \lambda_i(q) & \text{if } j = 1 \text{ for all } c \\ 4\lambda_i(q) & \text{if } j = 2, 3, 4, 5 \text{ and } c = I \\ -\lambda_i(q) & \text{if } j = 2, 3, 4, 5 \text{ and } c \neq I \end{cases}$$

From proposition (3.4)

$$\theta_{ij} = \sum_{\sigma \in \text{Gal}(Q(\lambda_{ij})/Q)} \sigma(\lambda_{ij})$$

$$\sigma \in \text{Gal}(Q(\lambda_{ij})/Q)$$

where  $\theta_{ij}$  is the rational valued of characters table of  $(Q_{2m} \times C_5)$  then

$$\theta_{ij}(u) = \sum_{\sigma \in \text{Gal}(Q(\lambda_{ij}(u))/Q)} \sigma(\lambda_{ij}(u)) = \sum_{\sigma \in \text{Gal}(Q(\lambda_{ij})/Q)} \sigma(\lambda_i(q) \cdot \lambda_j(c))$$

(I) if  $j = 1, \lambda_j(c) = 1$

$$\theta_{ij}(u) = \sum_{\sigma \in \text{Gal}(Q(\lambda_i(u))/Q)} \sigma(\lambda_i(q)) \cdot \lambda_j(c) = \theta_i(q) \cdot 1 = \theta_i(q) \cdot \theta_j(I)$$

where  $\theta_i$  is the rational valued of characters table of  $Q_{2m}$

(II) a - if  $j = 2, 3, 4, 5$  and  $c = I$

$$\theta_{ij}(u) = \sum_{\sigma \in \text{Gal}(Q(\lambda_{ij})/Q)} \sigma(\lambda_i(q) \cdot \lambda_j(I)) = \sum_{\sigma \in \text{Gal}(Q(\lambda_{ij})/Q)} \sigma(\lambda_i(q)) \cdot [\sum \sigma \lambda_j(I)]$$

$$= \sum_{\sigma \in \text{Gal}(Q(\lambda_{ij})/Q)} \sigma(\lambda_i(q)) [1+1+1+1] = \theta_i(q) \cdot 4 = \theta_i(q) \cdot \theta_j(I)$$

b - if  $j = 2, 3, 4, 5$  and  $c \neq I$

$$\theta_{ij}(u) = \sum_{\sigma \in \text{Gal}(Q(\lambda_{ij})/Q)} \sigma(\lambda_i(q) \cdot [\sum \sigma \lambda_j(c)])$$

$$= \sum_{\sigma \in \text{Gal}(Q(\lambda_{ij})/Q)} \sigma(\lambda_i(q)) [\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4]$$

$$= -\sum_{\sigma \in \text{Gal}(Q(\lambda_{ij})/Q)} \sigma(\lambda_i(q)) = \sum_{\sigma \in \text{Gal}(Q(\lambda_{ij})/Q)} \sigma(\lambda_i(q)) \cdot -1 = \theta_i(q) \cdot \theta_j(c)$$

From (I) and (II) we have  $\theta_{ij} = \theta_i \theta_j$  then

$$\equiv^* (Q_{2m} \times C_5) = \equiv^* (Q_{2m}) \times \equiv^* (C_5)$$

**Example (4.3)**

To find  $\equiv^* (Q_{14} \times C_5)$  by using theorem(4.2).from

$$\equiv^* (Q_{14}) =$$

$\Gamma$ -classes	[I]	$[x^2]$	$[x^7]$	[x]	[y]
$\theta_1$	1	1	1	1	1
$\theta_2$	6	-1	6	-1	0
$\theta_3$	1	1	1	1	-1
$\theta_4$	6	-1	-6	1	0
$\theta_5$	2	2	-2	-2	0

Table (8)

and  $\equiv^* (C_5) =$

$\Gamma$ -classes	[I]	[r]
$\theta_1$	1	1
$\theta_2$	4	-1

Table(9)

By theorem (4.2):

$$\equiv^* (Q_{14} \times C_5) = \equiv^* (Q_{14}) \times \equiv^* (C_5)$$

$$\equiv^* (Q_{14}) \times \equiv^* (C_5) =$$

$cl_\alpha$	[I, I]	[I, r]	[x <sup>2</sup> , I]	[x <sup>2</sup> , r]	[x <sup>7</sup> , I]	[x <sup>7</sup> , r]	[x, I]	[x, r]	[y, I]	[y, r]
$ cl_\alpha $	1	1	2	2	1	1	2	2	7	7
$\theta_{11}$	1	1	1	1	1	1	1	1	1	1
$\theta_{12}$	4	-1	4	-1	4	-1	4	-1	4	-1
$\theta_{21}$	6	6	-1	-1	6	6	-1	-1	0	0
$\theta_{22}$	24	-6	-4	1	24	-6	-4	1	0	0
$\theta_{31}$	1	1	1	1	1	1	1	1	-1	-1
$\theta_{32}$	4	-1	4	-1	4	-1	4	-1	-4	1
$\theta_{41}$	6	6	-1	-1	-6	-6	1	1	0	0
$\theta_{42}$	24	-6	-4	1	-24	6	4	-1	0	0
$\theta_{51}$	2	2	2	2	-2	-2	-2	-2	0	0
$\theta_{52}$	8	-2	8	-2	-8	2	-8	2	0	0

Table(10)

Which is same rational valued character table in example (4.1)

**Corollary(4.4):**

The general form of the rational valued characters table of the group  $Q_{2m} \times C_5$  is given by theorem (4.2)

**Proof:**

$$\equiv^* (Q_{2m}) =$$

$\Gamma$ -classes	[I]	[x <sup>2</sup> ]	[x <sup>m</sup> ]	[x]	[y]
$\theta_1$	1	1	1	1	1
$\theta_2$	m-1	-1	m-1	-1	0
$\theta_3$	1	1	1	1	-1
$\theta_4$	m-1	-1	-(m-1)	1	0
$\theta_5$	2	2	-2	-2	0

Table (11)

$$\equiv^* (C_5) =$$

$\Gamma$ -classes	[I]	[r]
$\theta_1$	1	1
$\theta_2$	4	-1

Table(12)

By theorem (4.2):

$$\equiv^* (Q_{2m} \times C_5) = \equiv^* (Q_{2m}) \times \equiv^* (C_5) =$$

$\Gamma$ -classes	[I, I]	[I, r]	[x <sup>2</sup> , I]	[x <sup>2</sup> , r]	[x <sup>m</sup> , I]	[x <sup>m</sup> , r]	[x, I]	[x, r]	[y, I]	[y, r]
$ cl_\alpha $	1	1	2	2	1	1	2	2	m	m
$\theta_{11}$	1	1	1	1	1	1	1	1	1	1
$\theta_{12}$	4	-1	4	-1	4	-1	4	-1	4	-1
$\theta_{21}$	m-1	m-1	-1	-1	m-1	m-1	-1	-1	0	0
$\theta_{22}$	4(m-1)	-(m-1)	-4	1	4(m-1)	-(m-1)	-4	1	0	0
$\theta_{31}$	1	1	1	1	1	1	1	1	-1	-1
$\theta_{32}$	4	-1	4	-1	4	-1	4	-1	-4	1
$\theta_{41}$	m-1	m-1	-1	-1	-(m-1)	-(m-1)	1	1	0	0
$\theta_{42}$	4(m-1)	-(m-1)	-4	1	-4(m-1)	m-1	4	-1	0	0
$\theta_{51}$	2	2	2	2	-2	-2	-2	-2	0	0
$\theta_{52}$	8	-2	8	-2	-8	2	-8	2	0	0

Table (13)

$\equiv (Q_{14 \times C_5}) =$

CL <sub>α</sub>	[I,I]	[I,r]	[I,r <sup>2</sup> ]	$\begin{bmatrix} I \\ r^3 \end{bmatrix}$	[I,r <sup>4</sup> ]	[x <sup>2</sup> ,I]	[x <sup>2</sup> ,r]	[x <sup>2</sup> ,r <sup>2</sup> ]	[x <sup>2</sup> ,r <sup>3</sup> ]	[x <sup>2</sup> ,r <sup>4</sup> ]	[x <sup>4</sup> ,I]	[x <sup>4</sup> ,r]	[x <sup>4</sup> ,r <sup>2</sup> ]	[x <sup>4</sup> ,r <sup>3</sup> ]	[x <sup>4</sup> ,r <sup>4</sup> ]	[x <sup>6</sup> ,I]	[x <sup>6</sup> ,r]	[x <sup>6</sup> ,r <sup>2</sup> ]	[x <sup>6</sup> ,r <sup>3</sup> ]	[x <sup>6</sup> ,r <sup>4</sup> ]	[x <sup>7</sup> ,I]	[x <sup>7</sup> ,r]	[x <sup>7</sup> ,r <sup>2</sup> ]	[x <sup>7</sup> ,r <sup>3</sup> ]	[x <sup>7</sup> ,r <sup>4</sup> ]	
Ψ <sub>(1,1)</sub>	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
Ψ <sub>(1,2)</sub>	1	ε	ε <sup>2</sup>	ε <sup>3</sup>	ε <sup>4</sup>	1	ε	ε <sup>2</sup>	ε <sup>3</sup>	ε <sup>4</sup>	1	ε	ε <sup>2</sup>	ε <sup>3</sup>	ε <sup>4</sup>	1	ε	ε <sup>2</sup>	ε <sup>3</sup>	ε <sup>4</sup>	1	ε	ε <sup>2</sup>	ε <sup>3</sup>	ε <sup>4</sup>	
Ψ <sub>(1,3)</sub>	1	ε <sup>2</sup>	ε <sup>4</sup>	ε	ε <sup>3</sup>	1	ε <sup>2</sup>	ε <sup>4</sup>	ε	ε <sup>3</sup>	1	ε <sup>2</sup>	ε <sup>4</sup>	ε	ε <sup>3</sup>	1	ε <sup>2</sup>	ε <sup>4</sup>	ε	ε <sup>3</sup>	1	ε <sup>2</sup>	ε <sup>4</sup>	ε	ε <sup>3</sup>	
Ψ <sub>(1,4)</sub>	1	ε <sup>3</sup>	ε	ε <sup>4</sup>	ε <sup>2</sup>	1	ε <sup>3</sup>	ε	ε <sup>4</sup>	ε <sup>2</sup>	1	ε <sup>3</sup>	ε	ε <sup>4</sup>	ε <sup>2</sup>	1	ε <sup>3</sup>	ε	ε <sup>4</sup>	ε <sup>2</sup>	1	ε <sup>3</sup>	ε	ε <sup>4</sup>	ε <sup>2</sup>	
Ψ <sub>(1,5)</sub>	1	ε <sup>4</sup>	ε <sup>3</sup>	ε <sup>2</sup>	ε	1	ε <sup>4</sup>	ε <sup>3</sup>	ε <sup>2</sup>	ε	1	ε <sup>4</sup>	ε <sup>3</sup>	ε <sup>2</sup>	ε	1	ε <sup>4</sup>	ε <sup>3</sup>	ε <sup>2</sup>	ε	1	ε <sup>4</sup>	ε <sup>3</sup>	ε <sup>2</sup>	ε	
χ <sub>(2,1)</sub>	2	2	2	2	2	V <sub>4</sub>	V <sub>4</sub>	V <sub>4</sub>	V <sub>4</sub>	V <sub>4</sub>	V <sub>6</sub>	V <sub>6</sub>	V <sub>6</sub>	V <sub>6</sub>	V <sub>6</sub>	V <sub>2</sub>	V <sub>2</sub>	V <sub>2</sub>	V <sub>2</sub>	V <sub>2</sub>	V <sub>2</sub>	2	2	2	2	2
χ <sub>(2,2)</sub>	2	2ε	2ε <sup>2</sup>	2ε <sup>3</sup>	2ε <sup>4</sup>	V <sub>4</sub>	εV <sub>4</sub>	ε <sup>2</sup> V <sub>4</sub>	ε <sup>3</sup> V <sub>4</sub>	ε <sup>4</sup> V <sub>4</sub>	V <sub>6</sub>	εV <sub>6</sub>	ε <sup>2</sup> V <sub>6</sub>	ε <sup>3</sup> V <sub>6</sub>	ε <sup>4</sup> V <sub>6</sub>	V <sub>2</sub>	εV <sub>2</sub>	ε <sup>2</sup> V <sub>2</sub>	ε <sup>3</sup> V <sub>2</sub>	ε <sup>4</sup> V <sub>2</sub>	2	2ε	2ε <sup>2</sup>	2ε <sup>3</sup>	2ε <sup>4</sup>	
χ <sub>(3)</sub>	2	2ε <sup>2</sup>	2ε <sup>4</sup>	2ε	2ε <sup>3</sup>	V <sub>4</sub>	ε <sup>2</sup> V <sub>4</sub>	ε <sup>4</sup> V <sub>4</sub>	εV <sub>4</sub>	ε <sup>3</sup> V <sub>4</sub>	V <sub>6</sub>	ε <sup>2</sup> V <sub>6</sub>	ε <sup>4</sup> V <sub>6</sub>	εV <sub>6</sub>	ε <sup>3</sup> V <sub>6</sub>	V <sub>2</sub>	ε <sup>2</sup> V <sub>2</sub>	ε <sup>4</sup> V <sub>2</sub>	εV <sub>2</sub>	ε <sup>3</sup> V <sub>2</sub>	2	2ε <sup>2</sup>	2ε <sup>4</sup>	2ε	2ε <sup>3</sup>	
χ <sub>(2,4)</sub>	2	2ε <sup>3</sup>	2ε	2ε <sup>4</sup>	2ε <sup>2</sup>	V <sub>4</sub>	ε <sup>3</sup> V <sub>4</sub>	εV <sub>4</sub>	ε <sup>4</sup> V <sub>4</sub>	ε <sup>2</sup> V <sub>4</sub>	V <sub>6</sub>	ε <sup>3</sup> V <sub>6</sub>	εV <sub>6</sub>	ε <sup>4</sup> V <sub>6</sub>	ε <sup>2</sup> V <sub>6</sub>	V <sub>2</sub>	ε <sup>3</sup> V <sub>2</sub>	εV <sub>2</sub>	ε <sup>4</sup> V <sub>2</sub>	ε <sup>2</sup> V <sub>2</sub>	2	2ε <sup>3</sup>	2ε	2ε <sup>4</sup>	2ε <sup>2</sup>	
χ <sub>(2,5)</sub>	2	2ε <sup>4</sup>	2ε <sup>3</sup>	2ε <sup>2</sup>	2ε	V <sub>4</sub>	ε <sup>4</sup> V <sub>4</sub>	ε <sup>3</sup> V <sub>4</sub>	ε <sup>2</sup> V <sub>4</sub>	εV <sub>4</sub>	V <sub>6</sub>	ε <sup>4</sup> V <sub>6</sub>	ε <sup>3</sup> V <sub>6</sub>	ε <sup>2</sup> V <sub>6</sub>	εV <sub>6</sub>	V <sub>2</sub>	ε <sup>4</sup> V <sub>2</sub>	ε <sup>3</sup> V <sub>2</sub>	ε <sup>2</sup> V <sub>2</sub>	εV <sub>2</sub>	2	2ε <sup>4</sup>	2ε <sup>3</sup>	2ε <sup>2</sup>	2ε	
χ <sub>(4,1)</sub>	2	2	2	2	2	V <sub>6</sub>	V <sub>6</sub>	V <sub>6</sub>	V <sub>6</sub>	V <sub>6</sub>	V <sub>2</sub>	V <sub>2</sub>	V <sub>2</sub>	V <sub>2</sub>	V <sub>2</sub>	V <sub>4</sub>	V <sub>4</sub>	V <sub>4</sub>	V <sub>4</sub>	V <sub>4</sub>	2	2	2	2	2	
χ <sub>(4,2)</sub>	2	2ε	2ε <sup>2</sup>	2ε <sup>3</sup>	2ε <sup>4</sup>	V <sub>6</sub>	εV <sub>6</sub>	ε <sup>2</sup> V <sub>6</sub>	ε <sup>3</sup> V <sub>6</sub>	ε <sup>4</sup> V <sub>6</sub>	V <sub>2</sub>	εV <sub>2</sub>	ε <sup>2</sup> V <sub>2</sub>	ε <sup>3</sup> V <sub>2</sub>	ε <sup>4</sup> V <sub>2</sub>	V <sub>4</sub>	εV <sub>4</sub>	ε <sup>2</sup> V <sub>4</sub>	ε <sup>3</sup> V <sub>4</sub>	ε <sup>4</sup> V <sub>4</sub>	2	2ε	2ε <sup>2</sup>	2ε <sup>3</sup>	2ε <sup>4</sup>	
χ <sub>(4,3)</sub>	2	2ε <sup>2</sup>	2ε <sup>4</sup>	2ε	2ε <sup>3</sup>	V <sub>6</sub>	ε <sup>2</sup> V <sub>6</sub>	ε <sup>4</sup> V <sub>6</sub>	εV <sub>6</sub>	ε <sup>3</sup> V <sub>6</sub>	V <sub>2</sub>	ε <sup>2</sup> V <sub>2</sub>	ε <sup>4</sup> V <sub>2</sub>	εV <sub>2</sub>	ε <sup>3</sup> V <sub>2</sub>	V <sub>4</sub>	ε <sup>2</sup> V <sub>4</sub>	ε <sup>4</sup> V <sub>4</sub>	εV <sub>4</sub>	ε <sup>3</sup> V <sub>4</sub>	2	2ε <sup>2</sup>	2ε <sup>4</sup>	2ε	2ε <sup>3</sup>	
χ <sub>(4,4)</sub>	2	2ε <sup>3</sup>	2ε	2ε <sup>4</sup>	2ε <sup>2</sup>	V <sub>6</sub>	ε <sup>3</sup> V <sub>6</sub>	εV <sub>6</sub>	ε <sup>4</sup> V <sub>6</sub>	ε <sup>2</sup> V <sub>6</sub>	V <sub>2</sub>	ε <sup>3</sup> V <sub>2</sub>	εV <sub>2</sub>	ε <sup>4</sup> V <sub>2</sub>	ε <sup>2</sup> V <sub>2</sub>	V <sub>4</sub>	ε <sup>3</sup> V <sub>4</sub>	εV <sub>4</sub>	ε <sup>4</sup> V <sub>4</sub>	ε <sup>2</sup> V <sub>4</sub>	2	2ε <sup>3</sup>	2ε	2ε <sup>4</sup>	2ε <sup>2</sup>	
χ <sub>(4,5)</sub>	2	2ε <sup>4</sup>	2ε <sup>3</sup>	2ε <sup>2</sup>	2ε	V <sub>6</sub>	ε <sup>4</sup> V <sub>6</sub>	ε <sup>3</sup> V <sub>6</sub>	ε <sup>2</sup> V <sub>6</sub>	εV <sub>6</sub>	V <sub>2</sub>	ε <sup>4</sup> V <sub>2</sub>	ε <sup>3</sup> V <sub>2</sub>	ε <sup>2</sup> V <sub>2</sub>	εV <sub>2</sub>	V <sub>4</sub>	ε <sup>4</sup> V <sub>4</sub>	ε <sup>3</sup> V <sub>4</sub>	ε <sup>2</sup> V <sub>4</sub>	εV <sub>4</sub>	2	2ε <sup>4</sup>	2ε <sup>3</sup>	2ε <sup>2</sup>	2ε	
χ <sub>(6,1)</sub>	2	2	2	2	2	V <sub>2</sub>	V <sub>2</sub>	V <sub>2</sub>	V <sub>2</sub>	V <sub>2</sub>	V <sub>4</sub>	V <sub>4</sub>	V <sub>4</sub>	V <sub>4</sub>	V <sub>4</sub>	V <sub>6</sub>	V <sub>6</sub>	V <sub>6</sub>	V <sub>6</sub>	V <sub>6</sub>	2	2	2	2	2	
χ <sub>(6,2)</sub>	2	2ε	2ε <sup>2</sup>	2ε <sup>3</sup>	2ε <sup>4</sup>	V <sub>2</sub>	εV <sub>2</sub>	ε <sup>2</sup> V <sub>2</sub>	ε <sup>3</sup> V <sub>2</sub>	ε <sup>4</sup> V <sub>2</sub>	V <sub>4</sub>	εV <sub>4</sub>	ε <sup>2</sup> V <sub>4</sub>	ε <sup>3</sup> V <sub>4</sub>	ε <sup>4</sup> V <sub>4</sub>	V <sub>6</sub>	εV <sub>6</sub>	ε <sup>2</sup> V <sub>6</sub>	ε <sup>3</sup> V <sub>6</sub>	ε <sup>4</sup> V <sub>6</sub>	2	2ε	2ε <sup>2</sup>	2ε <sup>3</sup>	2ε <sup>4</sup>	
χ <sub>(6,3)</sub>	2	2ε <sup>2</sup>	2ε <sup>4</sup>	2ε	2ε <sup>3</sup>	V <sub>2</sub>	ε <sup>2</sup> V <sub>2</sub>	ε <sup>4</sup> V <sub>2</sub>	εV <sub>2</sub>	ε <sup>3</sup> V <sub>2</sub>	V <sub>4</sub>	ε <sup>2</sup> V <sub>4</sub>	ε <sup>4</sup> V <sub>4</sub>	εV <sub>4</sub>	ε <sup>3</sup> V <sub>4</sub>	V <sub>6</sub>	ε <sup>2</sup> V <sub>6</sub>	ε <sup>4</sup> V <sub>6</sub>	εV <sub>6</sub>	ε <sup>3</sup> V <sub>6</sub>	2	2ε <sup>2</sup>	2ε <sup>4</sup>	2ε	2ε <sup>3</sup>	
χ <sub>(6,4)</sub>	2	2ε <sup>3</sup>	2ε	2ε <sup>4</sup>	2ε <sup>2</sup>	V <sub>2</sub>	ε <sup>3</sup> V <sub>2</sub>	εV <sub>2</sub>	ε <sup>4</sup> V <sub>2</sub>	ε <sup>2</sup> V <sub>2</sub>	V <sub>4</sub>	ε <sup>3</sup> V <sub>4</sub>	εV <sub>4</sub>	ε <sup>4</sup> V <sub>4</sub>	ε <sup>2</sup> V <sub>4</sub>	V <sub>6</sub>	ε <sup>3</sup> V <sub>6</sub>	εV <sub>6</sub>	ε <sup>4</sup> V <sub>6</sub>	ε <sup>2</sup> V <sub>6</sub>	2	2ε <sup>3</sup>	2ε	2ε <sup>4</sup>	2ε <sup>2</sup>	
χ <sub>(6,5)</sub>	2	2ε <sup>4</sup>	2ε <sup>3</sup>	2ε <sup>2</sup>	2ε	V <sub>2</sub>	ε <sup>4</sup> V <sub>2</sub>	ε <sup>3</sup> V <sub>2</sub>	ε <sup>2</sup> V <sub>2</sub>	εV <sub>2</sub>	V <sub>4</sub>	ε <sup>4</sup> V <sub>4</sub>	ε <sup>3</sup> V <sub>4</sub>	ε <sup>2</sup> V <sub>4</sub>	εV <sub>4</sub>	V <sub>6</sub>	ε <sup>4</sup> V <sub>6</sub>	ε <sup>3</sup> V <sub>6</sub>	ε <sup>2</sup> V <sub>6</sub>	εV <sub>6</sub>	2	2ε <sup>4</sup>	2ε <sup>3</sup>	2ε <sup>2</sup>	2ε	
Ψ <sub>(2,1)</sub>	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
Ψ <sub>(2,2)</sub>	1	ε	ε <sup>2</sup>	ε <sup>3</sup>	ε <sup>4</sup>	1	ε	ε <sup>2</sup>	ε <sup>3</sup>	ε <sup>4</sup>	1	ε	ε <sup>2</sup>	ε <sup>3</sup>	ε <sup>4</sup>	1	ε	ε <sup>2</sup>	ε <sup>3</sup>	ε <sup>4</sup>	1	ε	ε <sup>2</sup>	ε <sup>3</sup>	ε <sup>4</sup>	
Ψ <sub>(2,3)</sub>	1	ε <sup>2</sup>	ε <sup>4</sup>	ε	ε <sup>3</sup>	1	ε <sup>2</sup>	ε <sup>4</sup>	ε	ε <sup>3</sup>	1	ε <sup>2</sup>	ε <sup>4</sup>	ε	ε <sup>3</sup>	1	ε <sup>2</sup>	ε <sup>4</sup>	ε	ε <sup>3</sup>	1	ε <sup>2</sup>	ε <sup>4</sup>	ε	ε <sup>3</sup>	
Ψ <sub>(2,4)</sub>	1	ε <sup>3</sup>	ε	ε <sup>4</sup>	ε <sup>2</sup>	1	ε <sup>3</sup>	ε	ε <sup>4</sup>	ε <sup>2</sup>	1	ε <sup>3</sup>	ε	ε <sup>4</sup>	ε <sup>2</sup>	1	ε <sup>3</sup>	ε	ε <sup>4</sup>	ε <sup>2</sup>	1	ε <sup>3</sup>	ε	ε <sup>4</sup>	ε <sup>2</sup>	
Ψ <sub>(2,5)</sub>	1	ε <sup>4</sup>	ε <sup>3</sup>	ε <sup>2</sup>	ε	1	ε <sup>4</sup>	ε <sup>3</sup>	ε <sup>2</sup>	ε	1	ε <sup>4</sup>	ε <sup>3</sup>	ε <sup>2</sup>	ε	1	ε <sup>4</sup>	ε <sup>3</sup>	ε <sup>2</sup>	ε	1	ε <sup>4</sup>	ε <sup>3</sup>	ε <sup>2</sup>	ε	

Table ( 14 )





V <sub>1</sub>	V <sub>1</sub>	V <sub>1</sub>	V <sub>1</sub>	V <sub>1</sub>	V <sub>3</sub>	V <sub>3</sub>	V <sub>3</sub>	V <sub>3</sub>	V <sub>3</sub>	V <sub>5</sub>	V <sub>5</sub>	V <sub>5</sub>	V <sub>5</sub>	V <sub>5</sub>	0	0	0	0	0	0	0	0	0	
V <sub>1</sub>	εV <sub>1</sub>	ε <sup>2</sup> V <sub>1</sub>	ε <sup>3</sup> V <sub>1</sub>	ε <sup>4</sup> V <sub>1</sub>	V <sub>3</sub>	εV <sub>3</sub>	ε <sup>2</sup> V <sub>3</sub>	ε <sup>3</sup> V <sub>3</sub>	ε <sup>4</sup> V <sub>3</sub>	V <sub>5</sub>	εV <sub>5</sub>	ε <sup>2</sup> V <sub>5</sub>	ε <sup>3</sup> V <sub>5</sub>	ε <sup>4</sup> V <sub>5</sub>	0	0	0	0	0	0	0	0	0	
V <sub>1</sub>	ε <sup>2</sup> V <sub>1</sub>	ε <sup>4</sup> V <sub>1</sub>	εV <sub>1</sub>	ε <sup>3</sup> V <sub>1</sub>	V <sub>3</sub>	ε <sup>2</sup> V <sub>3</sub>	ε <sup>4</sup> V <sub>3</sub>	εV <sub>3</sub>	ε <sup>3</sup> V <sub>3</sub>	V <sub>5</sub>	ε <sup>2</sup> V <sub>5</sub>	ε <sup>4</sup> V <sub>5</sub>	εV <sub>5</sub>	ε <sup>3</sup> V <sub>5</sub>	0	0	0	0	0	0	0	0	0	
V <sub>1</sub>	ε <sup>3</sup> V <sub>1</sub>	εV <sub>1</sub>	ε <sup>4</sup> V <sub>1</sub>	ε <sup>2</sup> V <sub>1</sub>	V <sub>3</sub>	ε <sup>3</sup> V <sub>3</sub>	εV <sub>3</sub>	ε <sup>4</sup> V <sub>3</sub>	ε <sup>2</sup> V <sub>3</sub>	V <sub>5</sub>	ε <sup>3</sup> V <sub>5</sub>	εV <sub>5</sub>	ε <sup>4</sup> V <sub>5</sub>	ε <sup>2</sup> V <sub>5</sub>	0	0	0	0	0	0	0	0	0	
V <sub>1</sub>	ε <sup>4</sup> V <sub>1</sub>	ε <sup>3</sup> V <sub>1</sub>	ε <sup>2</sup> V <sub>1</sub>	εV <sub>1</sub>	V <sub>3</sub>	ε <sup>4</sup> V <sub>3</sub>	ε <sup>3</sup> V <sub>3</sub>	ε <sup>2</sup> V <sub>3</sub>	εV <sub>3</sub>	V <sub>5</sub>	ε <sup>4</sup> V <sub>5</sub>	ε <sup>3</sup> V <sub>5</sub>	ε <sup>2</sup> V <sub>5</sub>	εV <sub>5</sub>	0	0	0	0	0	0	0	0	0	
V <sub>3</sub>	V <sub>3</sub>	V <sub>3</sub>	V <sub>3</sub>	V <sub>3</sub>	V <sub>5</sub>	V <sub>5</sub>	V <sub>5</sub>	V <sub>5</sub>	V <sub>5</sub>	V <sub>1</sub>	V <sub>1</sub>	V <sub>1</sub>	V <sub>1</sub>	V <sub>1</sub>	0	0	0	0	0	0	0	0	0	
V <sub>3</sub>	εV <sub>3</sub>	ε <sup>2</sup> V <sub>3</sub>	ε <sup>3</sup> V <sub>3</sub>	ε <sup>4</sup> V <sub>3</sub>	V <sub>5</sub>	εV <sub>5</sub>	ε <sup>2</sup> V <sub>5</sub>	ε <sup>3</sup> V <sub>5</sub>	ε <sup>4</sup> V <sub>5</sub>	V <sub>1</sub>	εV <sub>1</sub>	ε <sup>2</sup> V <sub>1</sub>	ε <sup>3</sup> V <sub>1</sub>	ε <sup>4</sup> V <sub>1</sub>	0	0	0	0	0	0	0	0	0	
V <sub>3</sub>	ε <sup>2</sup> V <sub>3</sub>	ε <sup>4</sup> V <sub>3</sub>	εV <sub>3</sub>	ε <sup>3</sup> V <sub>3</sub>	V <sub>5</sub>	ε <sup>2</sup> V <sub>5</sub>	ε <sup>4</sup> V <sub>5</sub>	εV <sub>5</sub>	ε <sup>3</sup> V <sub>5</sub>	V <sub>1</sub>	ε <sup>2</sup> V <sub>1</sub>	ε <sup>4</sup> V <sub>1</sub>	εV <sub>1</sub>	ε <sup>3</sup> V <sub>1</sub>	0	0	0	0	0	0	0	0	0	
V <sub>3</sub>	ε <sup>3</sup> V <sub>3</sub>	εV <sub>3</sub>	ε <sup>4</sup> V <sub>3</sub>	ε <sup>2</sup> V <sub>3</sub>	V <sub>5</sub>	ε <sup>3</sup> V <sub>5</sub>	εV <sub>5</sub>	ε <sup>4</sup> V <sub>5</sub>	ε <sup>2</sup> V <sub>5</sub>	V <sub>1</sub>	ε <sup>3</sup> V <sub>1</sub>	εV <sub>1</sub>	ε <sup>4</sup> V <sub>1</sub>	ε <sup>2</sup> V <sub>1</sub>	0	0	0	0	0	0	0	0	0	
V <sub>3</sub>	ε <sup>4</sup> V <sub>3</sub>	ε <sup>3</sup> V <sub>3</sub>	ε <sup>2</sup> V <sub>3</sub>	εV <sub>3</sub>	V <sub>5</sub>	ε <sup>4</sup> V <sub>5</sub>	ε <sup>3</sup> V <sub>5</sub>	ε <sup>2</sup> V <sub>5</sub>	εV <sub>5</sub>	V <sub>1</sub>	ε <sup>4</sup> V <sub>1</sub>	ε <sup>3</sup> V <sub>1</sub>	ε <sup>2</sup> V <sub>1</sub>	εV <sub>1</sub>	0	0	0	0	0	0	0	0	0	
V <sub>5</sub>	V <sub>5</sub>	V <sub>5</sub>	V <sub>5</sub>	V <sub>5</sub>	V <sub>1</sub>	V <sub>1</sub>	V <sub>1</sub>	V <sub>1</sub>	V <sub>1</sub>	V <sub>3</sub>	V <sub>3</sub>	V <sub>3</sub>	V <sub>3</sub>	V <sub>3</sub>	0	0	0	0	0	0	0	0	0	
V <sub>5</sub>	εV <sub>5</sub>	ε <sup>2</sup> V <sub>5</sub>	ε <sup>3</sup> V <sub>5</sub>	ε <sup>4</sup> V <sub>5</sub>	V <sub>1</sub>	εV <sub>1</sub>	ε <sup>2</sup> V <sub>1</sub>	ε <sup>3</sup> V <sub>1</sub>	ε <sup>4</sup> V <sub>1</sub>	V <sub>3</sub>	εV <sub>3</sub>	ε <sup>2</sup> V <sub>3</sub>	ε <sup>3</sup> V <sub>3</sub>	ε <sup>4</sup> V <sub>3</sub>	0	0	0	0	0	0	0	0	0	
V <sub>5</sub>	ε <sup>2</sup> V <sub>5</sub>	ε <sup>4</sup> V <sub>5</sub>	εV <sub>5</sub>	ε <sup>3</sup> V <sub>5</sub>	V <sub>1</sub>	ε <sup>2</sup> V <sub>1</sub>	ε <sup>4</sup> V <sub>1</sub>	εV <sub>1</sub>	ε <sup>3</sup> V <sub>1</sub>	V <sub>3</sub>	ε <sup>2</sup> V <sub>3</sub>	ε <sup>4</sup> V <sub>3</sub>	εV <sub>3</sub>	ε <sup>3</sup> V <sub>3</sub>	0	0	0	0	0	0	0	0	0	
V <sub>5</sub>	ε <sup>3</sup> V <sub>5</sub>	εV <sub>5</sub>	ε <sup>4</sup> V <sub>5</sub>	ε <sup>2</sup> V <sub>5</sub>	V <sub>1</sub>	ε <sup>3</sup> V <sub>1</sub>	εV <sub>1</sub>	ε <sup>4</sup> V <sub>1</sub>	ε <sup>2</sup> V <sub>1</sub>	V <sub>3</sub>	ε <sup>3</sup> V <sub>3</sub>	εV <sub>3</sub>	ε <sup>4</sup> V <sub>3</sub>	ε <sup>2</sup> V <sub>3</sub>	0	0	0	0	0	0	0	0	0	
V <sub>5</sub>	ε <sup>4</sup> V <sub>5</sub>	ε <sup>3</sup> V <sub>5</sub>	ε <sup>2</sup> V <sub>5</sub>	εV <sub>5</sub>	V <sub>1</sub>	ε <sup>4</sup> V <sub>1</sub>	ε <sup>3</sup> V <sub>1</sub>	ε <sup>2</sup> V <sub>1</sub>	εV <sub>1</sub>	V <sub>3</sub>	ε <sup>4</sup> V <sub>3</sub>	ε <sup>3</sup> V <sub>3</sub>	ε <sup>2</sup> V <sub>3</sub>	εV <sub>3</sub>	0	0	0	0	0	0	0	0	0	
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	i	i	i	i	i	-i	-i	-i	-i	-i
-1	-ε	-ε <sup>2</sup>	-ε <sup>3</sup>	-ε <sup>4</sup>	-1	-ε	-ε <sup>2</sup>	-ε <sup>3</sup>	-ε <sup>4</sup>	-1	-ε	-ε <sup>2</sup>	-ε <sup>3</sup>	-ε <sup>4</sup>	i	iε	iε <sup>2</sup>	iε <sup>3</sup>	iε <sup>4</sup>	-i	-iε	-iε <sup>2</sup>	-iε <sup>3</sup>	-iε <sup>4</sup>
-1	-ε <sup>2</sup>	-ε <sup>4</sup>	-ε	-ε <sup>3</sup>	-1	-ε <sup>2</sup>	-ε <sup>4</sup>	-ε	-ε <sup>3</sup>	-1	-ε <sup>2</sup>	-ε <sup>4</sup>	-ε	-ε <sup>3</sup>	i	iε <sup>2</sup>	iε <sup>4</sup>	iε	iε <sup>3</sup>	-i	-iε <sup>2</sup>	-iε <sup>4</sup>	-iε	-iε <sup>3</sup>
-1	-ε <sup>3</sup>	-ε	-ε <sup>4</sup>	-ε <sup>2</sup>	-1	-ε <sup>3</sup>	-ε	-ε <sup>4</sup>	-ε <sup>2</sup>	-1	-ε <sup>3</sup>	-ε	-ε <sup>4</sup>	-ε <sup>2</sup>	i	iε <sup>3</sup>	iε	iε <sup>4</sup>	iε <sup>2</sup>	-i	-iε <sup>3</sup>	-iε	-iε <sup>4</sup>	-iε <sup>2</sup>
-1	-ε <sup>4</sup>	-ε <sup>3</sup>	-ε <sup>2</sup>	-ε	-1	-ε <sup>4</sup>	-ε <sup>3</sup>	-ε <sup>2</sup>	-ε	-1	-ε <sup>4</sup>	-ε <sup>3</sup>	-ε <sup>2</sup>	-ε	i	iε <sup>4</sup>	iε <sup>3</sup>	iε <sup>2</sup>	iε	-i	-iε <sup>4</sup>	-iε <sup>3</sup>	-iε <sup>2</sup>	-iε
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-i	-i	-i	-i	-i	I	i	i	i	I
-1	-ε	-ε <sup>2</sup>	-ε <sup>3</sup>	-ε <sup>4</sup>	-1	-ε	-ε <sup>2</sup>	-ε <sup>3</sup>	-ε <sup>4</sup>	-1	-ε	-ε <sup>2</sup>	-ε <sup>3</sup>	-ε <sup>4</sup>	-i	-iε	-iε <sup>2</sup>	-iε <sup>3</sup>	-iε <sup>4</sup>	I	iε	iε <sup>2</sup>	iε <sup>3</sup>	iε <sup>4</sup>
-1	-ε <sup>2</sup>	-ε <sup>4</sup>	-ε	-ε <sup>3</sup>	-1	-ε <sup>2</sup>	-ε <sup>4</sup>	-ε	-ε <sup>3</sup>	-1	-ε <sup>2</sup>	-ε <sup>4</sup>	-ε	-ε <sup>3</sup>	-i	-iε <sup>2</sup>	-iε <sup>4</sup>	-iε	-iε <sup>3</sup>	I	iε <sup>2</sup>	iε <sup>4</sup>	iε	iε <sup>3</sup>
-1	-ε <sup>3</sup>	-ε	-ε <sup>4</sup>	-ε <sup>2</sup>	-1	-ε <sup>3</sup>	-ε	-ε <sup>4</sup>	-ε <sup>2</sup>	-1	-ε <sup>3</sup>	-ε	-ε <sup>4</sup>	-ε <sup>2</sup>	-i	-iε <sup>3</sup>	-iε	-iε <sup>4</sup>	-iε <sup>2</sup>	I	iε <sup>3</sup>	iε	iε <sup>4</sup>	iε <sup>2</sup>
-1	-ε <sup>4</sup>	-ε <sup>3</sup>	-ε <sup>2</sup>	-ε	-1	-ε <sup>4</sup>	-ε <sup>3</sup>	-ε <sup>2</sup>	-ε	-1	-ε <sup>4</sup>	-ε <sup>3</sup>	-ε <sup>2</sup>	-ε	-i	-iε <sup>4</sup>	-iε <sup>3</sup>	-iε <sup>2</sup>	-iε	I	iε <sup>4</sup>	iε <sup>3</sup>	iε <sup>2</sup>	iε

Table (14)

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