

A NEW CLASS OF BH-ALGEBRA

صنف جديد من جبر إل- BH

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Abstract

In this paper, we introduce the notion of a U-BG-BH-algebra as a new class of a BH-algebra. Also we introduce the notion of a U-BG-ideal of U-BG-BH-algebra we state and prove some theorems and examples which determine the relationships between these notions and some types of ideals of a BH-algebra.

الخلاصة

عرفنا في هذا البحث مفهوم *U-BG-BH-algebra* كصنف جديد من جبر إل-BH, وكذلك قدمنا مفهوم U-BG-ideal في جبر U-BG-BH-algebra. وأعطينا وبرهنا بعض المبرهنات والأمثلة التي تحدد العلاقة بين هذه الأفكار مع بعض أنواع المثاليات في جبر إل-BH.

1. INTRODUCTION

The notion of BCK-algebra and BCI-algebra was formulated first in 1966 by Y.Imai and K.Iseki as a generalization of the concept of set-theoretic difference and propositional calculus [7]. In 1983, Q.P.Hu and X. Li introduced the notion of a BCH-algebra which was a generalization of BCK/BCI-algebras [9]. In 1998, Y. B. Jun, E. H. Roh and H .S. Kim introduced the notion of BH-algebra, which is a generalization of BCH-algebra and the notion of ideal [12]. In 2002, j.negggers and Hee Sik Kim introduced the notion of B-algebra[5]. In 2008, Chang Bum Kim and Hee Sik Kim introduced the notion of BG-algebra which is a generalization of B-algebras [1]. In 2010 K.Megalai and A.Tamilarasi introduced the notion of TM-algebra [8]. In 2011 H. H. Abass and H. M. Saeed generalized the notion of BCA-part to BH-algebra[4]. In 2012 H.H. Abass and H. B. Dahham generalized the notion of BH-ideal to BH-algebra[3].

2. PRELIMINARIES

In this section, we give some basic concepts about a B-algebra, BG-algebra, BCK-algebra, BCI-algebra, BCH-algebra, TM-algebra, BH-algebra, (subalgebra, BCA-part, homomorphism, ideal, BH-ideal) of a BH-algebra with some theorems, propositions and examples.

Definition (2.1):[5]

A **B-algebra** is a non empty set X with a constant 0 and a binary operation "*" satisfying the following axioms:

- i. $x*x=0$
- ii. $x*0=x$
- iii. $(x*y)*z = x*(z*(0*y))$, for all $x,y,z \in X$.

Definition (2.2) :[1]

A **BG-algebra** is a non-empty set X with a constant 0 and a binary operation " $*$ " satisfying the following axioms:

- i. $x*x=0$
- ii. $x*0=x$
- iii. $(x*y)*(0*y)=x$, for all $x,y \in X$.

Lemma (2.3) :[1]

Let $(X,*,0)$ be a BG-algebra. Then

- i. The right cancellation law holds in X , i.e., $x*y = z*y$ implies $x=z$
- ii. $0*(0*x)=x$, for all $x \in X$
- iii. If $x*y=0$, then $x=y$ for all $x,y \in X$,
- iv. If $0*x=0*y$, then $x=y$ for all $x,y \in X$,
- v. $(x*(0*x))*x=x$, for all $x \in X$.

Definition (2.4) :[10]

A **BCI-algebra** is an algebra $(X,*,0)$ of type $(2,0)$, where X is a nonempty set, " $*$ " is a binary operation and 0 is a constant, satisfying the following axioms:

- i. $((x*y)*(x*z))*(z*y) = 0$, for all $x, y, z \in X$,
- ii. $(x*(x*y))*y = 0$, for all $x, y \in X$,
- iii. $x * x = 0$, for all $x \in X$,
- iv. $x * y = 0$ and $y * x = 0$ imply $x = y$, for all $x, y \in X$.

Definition (2.5) :[6]

A **BCK-algebra** is a BCI-algebra satisfying the axiom: $0 * x = 0$, for all $x \in X$.

Definition (2.6):[9]

A **BCH-algebra** is an algebra $(X,*,0)$, where X is a nonempty set, " $*$ " is a binary operation and 0 is a constant, satisfying the following axioms:

- i. $x*x=0, \forall x \in X$.
- ii. $x*y=0$ and $y*x=0$ imply $x=y, \forall x,y \in X$.
- iii. $(x*y)*z=(x*z)*y, \forall x,y,z \in X$.

Definition(2.7):[8]

A **TM-algebra** $(X,*,0)$ is a non-empty set X with a constant 0 and a binary operation satisfying the following axioms :

- i. $x*0 = x$, for all $x \in X$.
- ii. $(x * y) (x * z) = z *y$, for all $x, y, z \in X$.

Definition (2.8) :[2]

A **BH-algebra** is a nonempty set X with a constant 0 and a binary operation " $*$ " satisfying the following conditions:

- i. $x*x=0, \forall x \in X$.
- ii. $x*y=0$ and $y*x=0$ imply $x = y, \forall x, y \in X$.
- iii. $x*0 =x, \forall x \in X$.

Remark (2.9):[12]

- i. Every BCK-algebra is a BCH-algebra.
- ii. Every BCK-algebra is a BH-algebra.
- iii. Every BCI-algebra is a BH-algebra.

Definition (2.10) :[2]

Let X a BH-algebra and $S \subseteq X$. Then S is called a *subalgebra* of X if $x*y \in S$ for all $x,y \in S$.

Definition (2.11) :[4]

Let X be a BH-algebra. Then the set $X_+ = \{x \in X : 0*x=0\}$ is called the *BCA-part* of X .

Definition (2.12) [3] :

A non-empty subset N of a BH-algebra X is said to be *normal* of X if $(x * a) *(y * b) \in N$, for all $x * y, a * b \in N$.

Theorem (2.13) [3].

Every normal subset N of a BH-algebra X is a subalgebra of X .

Remark(2.14):[11]

Let X and Y be BH-algebras. A mapping $f:X \rightarrow Y$ is called a homomorphism if $f(x*y)=f(x)*f(y)$ for all $x,y \in X$. a homomorphism f is called a monomorphism (resp., epimorphism) if it is injective (resp., surjective). A bijective homomorphism is called an isomorphism. Two BH-algebras X and Y are said to be isomorphic, written $X \cong Y$, if there exists an isomorphism $f:X \rightarrow Y$. For all homomorphism $f:X \rightarrow Y$, the set $\{x \in X : f(x)=0\}$ is called the kernel of f , denoted by $\text{Ker}(f)$, and the set $\{f(x) : x \in X\}$ is called the image of f , denoted by $\text{Im}(f)$. Notice that $f(0)=0'$, for all homomorphism f .

Definition (2.15):[8]

Let $f : X \rightarrow Y$ be a *homomorphism of a TM-algebras*. Then $f^{-1}(Y) = \{ x \in X : f(x) = y, \text{ for some } y \in Y \}$ and $f(X) = \{ f(x) : x \in X \}$ is called the image of f .

We generalize the concept $f^{-1}(Y)$ to a *BH-algebra*

Definition (2.16):

Let $f : X \rightarrow Y$ be a *homomorphism of a BH-algebras*. Then $f^{-1}(Y) = \{ x \in X : f(x) = y, \text{ for some } y \in Y \}$ and $f(X) = \{ f(x) : x \in X \}$ is called the image of f .

Definition (2.17) :[12]

Let I be a nonempty subset of a BH-algebra X . Then I is called an *ideal* of X if it satisfies:

- i. $0 \in I$.
- ii. $x*y \in I$ and $y \in I$ imply $x \in I$.

Definition (2.18) :[3]

Let X be a BH-algebra and I be a subset of X . Then I is called a *BH-ideal* of X if it satisfies following conditions:

- i. $0 \in I$,
- ii. $x*y \in I$ and $y \in I \Rightarrow x \in I$,
- iii. $x \in I$ and $y \in X \Rightarrow x*y \in I, I*X \subseteq I$.

3. THE MAIN RESULTS

In this section, we introduce the concepts a U-BG-BH-algebra, a U-BG-ideal of U-BG-BH-algebra. Also we state and prove some theorems and examples about these concepts.

Definition (3.1):

A *BH-algebra* X is called is a *U-BG-BH-algebra* if there exists a proper subset U of X , such that:

- i. $0 \in U, |U| \geq 2$.
- ii. U is a BG-algebra.

Example (3.2):

Consider the BH-algebra $X=\{0,1,2,3\}$ with binary operation "*" defined as follows:

| | | | | |
|----------|----------|----------|----------|----------|
| * | 0 | 1 | 2 | 3 |
| 0 | 0 | 1 | 2 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 2 | 2 | 2 | 0 | 0 |
| 3 | 3 | 2 | 1 | 0 |

The subset $U=\{0,1,2\}$ of X is a BG-algebra in X , with the binary operation "*", Therefore X is a U-BG-BH algebra.

Lemma (3.3):

If X is an U-BG-BH algebra, then X is a BH-algebra.

Proof:

Is from definition (3.1). ■

Remark (3.4):

The converse of lemma (3.3) is not correct in general as in the following example.

Example (3.5):

Let $X=\{0,1,2\}$ be a set with binary operation "*" defined by:

| | | | |
|----------|----------|----------|----------|
| * | 0 | 1 | 2 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 |
| 2 | 2 | 1 | 0 |

Then X is a BH- algebra but it is not U-BG-BH-algebra, because X is not contain a proper subset U , such that U is a BG-algebra.

Proposition (3.6):

Let X be a BH algebra and $X= X_+$. Then X is not U-BG-BH algebra.

Proof:

Let $U \subseteq X$ is a proper BG-algebra and let $0, x \in U$ such that $x \neq 0$
 $\Rightarrow 0*x=0=0*0$. [Since $X= X_+$. By definition (2.11)]
 $\Rightarrow U$ is not BG-algebra because $0*x=0*0 \Rightarrow x=0$. [By lemma(2.3)(iv)]
 This contradiction!
 $\Rightarrow X$ is not U-BG-BH algebra. ■

Corollary (3.7):

Let X be an BCK-algebra. Then X is a not a U-BG-BH-algebra.

Proof:

Let X be a BCK-algebra, since every BCK-algebra is a BH-algebra [By. Remark(2.9)(ii)]
 $\Rightarrow 0*x=0$ [By. Definition (2.5)]
 $\Rightarrow X=X_+$.
 Therefore, X is not a U-BG-BH algebra. ■ [By. Proposition(3.6)]

Definition (3.8):

A nonempty subset I of a U-BG-BH algebra X is called a ***U-BG-ideal*** of X related to U if it satisfies:

- i. $0 \in I$.
- ii. $x*y \in I \Rightarrow x \in I$, for all $x \in U$ and $y \in I$.

Example (3.9):

Consider a U-BG-BH-algebra $X=\{0,1,2,3\}$ with binary operation "*" defined as follows:

| | | | | |
|----------|----------|----------|----------|----------|
| * | 0 | 1 | 2 | 3 |
| 0 | 0 | 1 | 2 | 0 |
| 1 | 1 | 0 | 1 | 2 |
| 2 | 2 | 2 | 0 | 1 |
| 3 | 3 | 1 | 0 | 0 |

where $U=\{0,1,2\}$. The subset $I=\{0,3\}$ of X is U-BG-ideal of X .

Remark (3.10):

$\{0\}, X$ are U-BG- ideals of X , they are called trivial U-BG-ideals. An ideal I is said to be a proper if $I \neq X$.

Proposition (3.11):

Let X be a U-BG-BH-algebra. Then every ideal of X is a U-BG-ideal of X .

Proof:

Let I be an ideal of X , $x \in U$ and $y \in I$ such that $x*y \in I$,

$\Rightarrow x \in I$,

[Since $U \subseteq X$ and I is an ideal of X]

$\Rightarrow I$ is a U-BG-ideal of X . ■

Remark (3.12):

The converse of Proposition (3.11) is not correct in general as in the following example.

Example (3.13):

Consider the U-BG-BH-algebra X in example (3.9), where $U=\{0,1,2\}$, the subset $I=\{0,2\}$ is a U-BG-ideal of X , but I is not an ideal of X . Since $3 \in X$, $2 \in I$ and $3*2=0 \in I$ but $3 \notin I$.

Proposition (3.14):

Let X be a U-BG-BH-algebra, U_1 and U_2 be a BG- algebra ,which are properly contained in X and $U_1 \subseteq U_2$, Then every U_2 -BG-ideal is U_1 -BG-ideal.

Proof:

Let I be U_2 -BG-ideal and let $x \in U_1$ and $y \in I$ such that $x*y \in I$

$\Rightarrow x \in I$.

[Since $U_1 \subseteq U_2$ and U_2 -BG-ideal of X]

$\Rightarrow I$ is U_1 -BG-ideal. ■

Remark (3.15):

The converse of Proposition (3.14) is not correct in general as in the following example.

Example (3.16):

Consider the U-BG-BH-algebra $X=\{0,1,2,3,4\}$ with binary operation "*" defined as follows.

| | | | | | |
|----------|----------|----------|----------|----------|----------|
| * | 0 | 1 | 2 | 3 | 4 |
| 0 | 0 | 1 | 2 | 3 | 0 |
| 1 | 1 | 0 | 3 | 2 | 0 |
| 2 | 2 | 3 | 0 | 1 | 0 |
| 3 | 3 | 2 | 1 | 0 | 0 |
| 4 | 4 | 3 | 1 | 2 | 0 |

The subset $U_1=\{0,1\}$ and $U_2=\{0,1,2,3\}$ are BG-algebra which are properly contained in X and $U_1 \subseteq U_2$, the subset $I=\{0,1,2\}$ is U_1 -BG-ideal of X and it is not a U_2 -BG-ideal of X . Since $3 \in U_2, 1 \in I$ such that $3*1=2 \in I$ but $3 \notin I$.

Proposition (3.17):

Let X be a U-BG-BH-algebra. Then every proper subset of X contain U is a U-BG-ideal of X .

Proof:

Let X be a U-BG-BH-algebra and I be subset of X contain U , to prove I is a U-BG-ideal of X .

Let $x \in U$ and $y \in I$ such that $x*y \in I$

$\Rightarrow x \in I$,

[Since $U \subseteq I$]

$\Rightarrow I$ is a U-BG-ideal. ■

Theorem (3.18):

Let X be a U-BG-BH-algebra . Then every normal subalgebra is a U-BG- ideal of X .

Proof:

Let X be a U-BG- BH-algebra and N be a normal subalgebra, to prove N is a U-BG-ideal of X .

Let $x, y \in N$. then $x * y \in N$.

[Since N is a normal subalgebra. By theorem(2.13)]

i. $0=(x * y) *(x * y) \in N$

ii. Let $x \in U, x*y \in N$ and $y \in N$

Since $x*y, 0*y \in N$,

$\Rightarrow (x*y)*(0*y) \in N$

[Since N is a normal subalgebra. By theorem(2.13)]

$\Rightarrow (x*0)*(y*y) \in N$,

$\Rightarrow x*0 \in N$

[Since $x*0=x$]

$\Rightarrow x \in N$,

$\Rightarrow N$ is a U-BG- ideal of X . ■

Proposition (3.19):

Let X be a U-BG-BH-algebra and $\{ I_i, i \in \lambda \}$ be a family of a U-BG-ideals of X . Then $\bigcap_{i \in \lambda} I_i$ is a U-BG-ideal of X .

Proof:

i. Since $0 \in I_i \quad \forall i \in \lambda$
definition(3.8)(i)]

[Since I_i is a U-BG-ideal , $\forall i \in \lambda$. By

$\Rightarrow 0 \in \bigcap_{i \in \lambda} I_i$

ii. Let $x \in U$ such that $x*y \in \bigcap_{i \in \lambda} I_i, y \in \bigcap_{i \in \lambda} I_i$

$\Rightarrow x*y \in I_i$ and $y \in I_i, (\forall i \in \lambda),$

$\Rightarrow x \in I_i, (\forall i \in \lambda)$

[Since I_i U-BG-ideal of $X, \forall i \in \lambda$,by

definition(3.8)(ii)]

$\Rightarrow x \in \bigcap_{i \in \lambda} I_i,$

$\Rightarrow \bigcap_{i \in \lambda} I_i$ is a U-BG-ideal of X . ■

Remark (3.20):

Let X be a U-BG-BH-algebra and I, J be a U-BG ideal of X . Then $I \cup J$ is not necessary be a U-BG ideal of X as in the following example.

Example (3.21):

Consider the U-BG-BH-algebra $X=\{0,1,2,3,4\}$ with binary operation "*" defined as follows.

| * | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 0 | 0 |
| 1 | 1 | 0 | 1 | 4 | 3 |
| 2 | 2 | 2 | 0 | 1 | 1 |
| 3 | 3 | 1 | 2 | 0 | 2 |
| 4 | 4 | 3 | 1 | 2 | 0 |

where $U=\{0,1,2\}$, $I=\{0,3\}$ and $J=\{0,4\}$ are two U-BG-ideals of X, but $I \cup J =\{0,3,4\}$ is not a U-BG-ideal of X. Since $1 \in U, 3 \in I \cup J$ and $1*3=4 \in I \cup J$ but $1 \notin I \cup J$.

Proposition (3.22):

Let X be a U-BG-BH-algebra and $\{ I_i , i \in \lambda \}$ be a chain of a U-BG-ideals of X. Then $\bigcup_{i \in \lambda} I_i$ is a

U-BG-ideal of X.

Proof:

i. Since $0 \in I_i , \forall i \in \lambda$ [Since I_i is a U-BG-ideal of X, $\forall i \in \lambda$, by definition(3.8)(i)]

$$\Rightarrow 0 \in \bigcup_{i \in \lambda} I_i$$

ii. Let $x \in U$ such that $x*y \in \bigcup_{i \in \lambda} I_i , y \in \bigcup_{i \in \lambda} I_i$

$\Rightarrow \exists I_i , I_k \in \{ I_i \}_{i \in \lambda}$, such that $x*y \in I_i$ and $y \in I_k$,

\Rightarrow either $I_i \subseteq I_k$ or $I_k \subseteq I_i$,

If $I_i \subseteq I_k$

$\Rightarrow x*y \in I_k$ and $y \in I_k$,

$\Rightarrow x \in I_i$

[Since I_i is a U-BG-ideal of X, $\forall i \in \lambda$, by definition(3.8)(i)]

$$\Rightarrow x \in \bigcup_{i \in \lambda} I_i ,$$

Similarly, If $I_k \subseteq I_i$,

therefore $\bigcup_{i \in \lambda} I_i$ is a U-BG-ideal of X. ■

Theorem (3.23):

Let X be U-BG-BH-algebra and I be a U-BG- ideal such that $x*y \notin I$, for all $x \notin I$ and $y \in I$. Then I is an ideal of X.

Proof:

Let I be a U-BG-ideal of X , $x \in X$ and $y \in I$.

i. $0 \in I$

[Since I is a U-BG-ideal of X. By definition(3.8)(i)]

ii. Let $x*y \in I$ and $y \in I$

\Rightarrow There are two cases.

Case(I): If $x \in U \Rightarrow x \in I$

[Since I is a U-BG-ideal of X. By definition(3.8)(ii)]

Case(II): If $x \notin U$, either $x \in I$ or $x \notin I$,

If $x \in I \Rightarrow I$ is an ideal,

If $x \notin I \Rightarrow x*y \notin I$,

[By hypothesis]

and this contradiction! since $x*y \in I$,

Therefore, I is an ideal of X. ■

Proposition (3.24):

Let $f:(X,*,0) \rightarrow (Y,*/',0')$ be a U-BG-BH-epimorphism, such that $f(U)$ is a BG-algebra of X , if I is a U-BG-ideal of X . Then $f(I)$ is a $f(U)$ -BG-ideal of Y .

Proof:

Let I be a U-BG-ideal of X .

- i. Since $0 \in I \Rightarrow f(0) = 0' \in f(I)$ [Since I is a U-BG-ideal of X . By definition(3.8)(i)]
- ii. Let $x \in f(U)$ such that $x *' y \in f(I)$ and $y \in f(I)$
 $\Rightarrow \exists a \in U, b \in I$ such that $f(a) = x, f(b) = y,$
 $\Rightarrow f(a) *' f(b) \in f(I)$ and $f(b) \in f(I),$
 $\Rightarrow f(a * b) \in f(I)$ and $f(b) \in f(I),$
 $\Rightarrow a * b \in I$ and $b \in I,$
 $\Rightarrow a \in I$ [Since I is a U-BG-ideal of X . By definition(3.8)(ii)]
 $\Rightarrow f(a) \in f(I)$
 $\Rightarrow x \in f(I).$
 $\Rightarrow f(I)$ is a U-BG-ideal of X . ■

Proposition (3.25):

Let $f:(X,*,0) \rightarrow (Y,*/',0')$ be a U-BG-BH-epimorphism, such that $f^{-1}(U)$ is a BG-algebra of X , if I is a U-BG-ideal of Y . Then $f^{-1}(I)$ is a $f^{-1}(U)$ -BG-ideal in X .

Proof:

Let I be an U-BG-ideal in Y

- i. Since $f(0) = 0' \Rightarrow 0 \in f^{-1}(I)$ [Since I is a U-BG-ideal of X . By definition(3.8)(i)]
- ii. Let $x \in f^{-1}(U)$ such that $x *' y \in f^{-1}(I)$ and $y \in f^{-1}(I)$
 $\Rightarrow f(x *' y) \in I$ and $f(y) \in I$
 $\Rightarrow f(x) *' f(y) \in I$ and $f(y) \in I$
 $\Rightarrow f(x) \in I$ [Since I is a U-BG-ideal of X . By definition(3.8)(ii)]
 $\Rightarrow x \in f^{-1}(I)$
 $\Rightarrow f^{-1}(I)$ is a U-BG-ideal of X . ■

Theorem (3.26):

Let X be a U-BG-BH-algebra. Then every U-BG-ideal in U is a BG-algebra with the same binary operation on X and the constant 0.

Proof:

Let X be a U-BG-BH-algebra, and I be a U-BG-ideal of X .

- i. Let $x \in I \Rightarrow x \in U \Rightarrow x * x = 0$ [Since U is a BG-algebra. By definition(2.2)(i)]
- ii. Let $x \in I \Rightarrow x \in U \Rightarrow x * 0 = x$ [Since U is a BG-algebra. By definition(2.2)(ii)]
- iii. Let $x, y \in I, \Rightarrow x, y \in U \Rightarrow (x * y) * (0 * y) = x$ [Since U is a BG-algebra. By definition(2.2)(iii)]
 $\Rightarrow I$ is a BG-algebra of X . ■

Remark (3.27):

Let X be a U-BG-BH-algebra and I be a U-BG ideal of X . Then I is not necessary a BH-algebra as in the following example.

Example (3.28):

Consider the U-BG-BH algebra $X=\{0,1,2,3\}$ with binary operation "*" defined as follows.

| * | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 2 |
| 1 | 1 | 0 | 1 | 2 |
| 2 | 2 | 2 | 0 | 1 |
| 3 | 3 | 2 | 1 | 0 |

where $U=\{0,1,2\}$, the subset $I=\{0,3\}$ of X is U-BG-ideal but not BH-algebra, since I is not closed under "*".

Proposition (3.29):

Let X be a U-BG-BH algebra. Then every BH-ideal is a U-BG- ideal of X .

Proof:

Let X be a U-BG- BH algebra ,and I be a BH-ideal of X ,
 Since every BH-ideal is an ideal of X and every ideal is U-BG-ideal of X . [By proposition(3.11)]
 $\Rightarrow I$ is a U-BG-ideal of X . ■

Remark (3.30):

The converse of proposition(3.29) is not correct in general as in the following example.

Example (3.31):

Consider the U-BG-BH-algebra X in example(3.9), where $U=\{0,1,2\}$, the subset $I=\{0,1\}$ of X is U-BG-ideal of X which is not a BH-ideal of X , Since $0 \in I, 2 \in X$ but $0*2=2 \notin I$.

Proposition (3.32):

Let X be a U-BG-BH algebra. Then every U-BG- ideal of X satisfies the condition $(z * y)* x = 0$ then $z \in I$, for all $x,y \in I, z \in U$.

Proof:

Let $x,y \in I, z \in U$ and $(z*y)*x=0 \in I$,
 $\Rightarrow (z*y)*x \in I, x \in I$,
 $\Rightarrow (z*y) \in I$ [Since I is a U-BG-ideal]
 $\Rightarrow (z*y) \in I, y \in I$
 $\Rightarrow z \in I$. ■ [Since I is a U-BG-ideal]

Proposition (3.33):

Let X and Y be two U-BG-BH-algebra and $f:X \rightarrow Y$ be a BH-homomorphism. Then $\ker(f)$ is a U-BG- ideal.

Proof:

i. Since $f(0)=0' \Rightarrow 0 \in \ker(f)$
 ii. Let $x \in U$ such that $x*y \in \ker(f)$ and $y \in \ker(f)$
 $\Rightarrow 0=f(x*y)=f(x)*f(y)=f(x)*0=f(x)$
 $\Rightarrow x \in \ker(f)$
 $\Rightarrow \ker(f)$ is an U-BG-ideal of X . ■

Proposition (3.34):

Let X be a U-BG-BH algebra and I be a U-BG-ideal of X such that if $x*y=0 \in I$. Then $x \in I$, for all $x \in U$ and $y \in I$.

Proof:

Let $x \in U$ and $y \in I \Rightarrow x*y=0 \in I$
 $\Rightarrow x*y \in I, y \in I \Rightarrow x \in I$. ■ [Since I is U-BG- ideal]

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