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THE COMBINED TEMIMI TRANSFORM-DIFFERENTIAL TRANSFORM METHOD FOR SOLVING LINEAR NON-HOMOGENEOUS PDEs

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ABSTRACT

In this work, a combined form of Temimi transform method (TTM) with the Differential transform method (DTM) will be used to solve non-homogeneous linear partial differential equations (PDEs). The combined method is capable of handling non-homogeneous linear partial differential equations with variable coefficient.

1- INTRODUCTON

Many interesting phenomena in scientific and engineering applications are governed differential equations, employed to model innumerable nonlinear phenomena, for instance in solid state physics, plasma physics, fluid dynamics, mathematical biology and chemical kinetics. The wave equation, heat equation and laplace equation are known as three fundamental equations in mathematical physics, and occur in many branches of physics, in applied mathematics as well as in engineering. It is also known that there are two types of these equations, the homogeneous equations that have constant coefficients with many classical solutions, and the non-homogeneous equations with constant coefficients. Most of these types of equations do not have an analytical solution, these equations should be solved by using numerical or approximate methods. In the last decade, there has been some advanced developments including, Adomian decomposition method, Differential transform method, Variational iteration method and Homotopy perturbation method for solving various types of partial differential equations(PDEs).

There are several researchers have worked in this field such as Fashid Mirzeaa who used (DTM) to applied linear and non linear system on (O.D.E) and the other is Javed Ali who worked on the boundary values problems in a finite domain with two point boundary conditions .

The basic motivation of this work, is to propose anew modification of Differential transform method (DTM), which is based on a combination of the Temimi transform and the Differential transform methods, called (TDTM), that will be used together to establish exact solutions, or approximations of homogeneous and non-homogeneous partial differential equations.

2- TEMIMI TRANSFORM METHOD (TTM) see[4]

Definition1:

AL-Temimi transformation for the function $u(x,t)$, $(t > 1)$ is defined

$$T\{u(x,t)\} = \int_1^\infty t^{-s} u(x,t)dt = v(x,s),$$

by condition the integral is convergent, s is constant.

The Temimi transform (TTM) of some fundamental functions are given in table (1).

ID	FUNCTIONS, $f(x)$	$F(s) = \int_1^\infty x^{-s} f(x)dx = T\{f(x)\}$	Regional of convergence
1	$k, k\ is\ constant$	$\frac{k}{s-1}$	$s > 1$
2	$x^n, n \in R$	$\frac{1}{s-(n+1)}$	$s > n+1$
3	lnx	$\frac{1}{(s-1)^2}$	$s > 1$
4	$x^n lnx, n \in R$	$\frac{1}{(s-(n+1))^2}$	$s > 1$
5	$\sin(alnx)$	$\frac{a}{(s-1)^2 + a^2}$	$s > 1$
6	$\cos(alnx)$	$\frac{s-1}{(s-1)^2 + a^2}$	$s > 1$
7	$\sinh(alnx)$	$\frac{a}{(s-1)^2 - a^2}$	$\lvert s-1 \rvert > a$
8	$\cosh(alnx)$	$\frac{s-1}{(s-1)^2 - a^2}$	$\lvert s-1 \rvert > a$

Table 1 : Transform of some command functions.

This method is solve the PDEs with variable coefficients (Euler's equation) .

Definition 2

Let $u(x,t)$ be a function where $(t > 1)$ and $T\{u(x,t)\} = v(x,s)$, $u(x,t)$ is said to be an inverse for the Temimi transformation and written as $T^{-1}\{v(x,s)\} = u(x,t)$ where T^{-1} returns the transformation to the original function.

For example $T^{-1}\left(\frac{x}{s-3}\right) = xt^2$, by using table (1) (ID-2).

TEOREM 1

If $T\{f(x)\} = F(s)$ and a is constant, then $T\{x^{-a} f(x)\} = F(s+a)$.

TEOREM 2

If $u(x,t)$ has the Temimi transformation $v(x,s)$ such that $u(x,t)$ is define for all $(t > 1)$ then :

1- $T\{tu_t(x,t)\} = -u(x,1) + (s-1)v(x,s)$
2- $T\{t^2 u_{tt}(x,t)\} = -u_t(x,1) - (s-2)u(x,1) + (s-2)(s-1)v(x,s)$
3- $T\{xu_x(x,t)\} = x\frac{d}{dx}v(x,s)$
4- $T\{x^2 u_{xx}(x,t)\} = x^2\frac{d^2}{dx^2}v(x,s).$

3- DIFFERENTIAL TRANSFORM METHOD (DTM) see[1] ,[5]

The transformation of the kth derivative of a function in on variable is a following :

$$Y(k) = \frac{1}{k!}\left[\frac{d^k y}{dx^k}(x)\right]_{x=x_0} , \qquad (1)$$

and the inverse transformation is defined by ,

$$y(x) = \sum_{k=0}^{\infty} Y(k)(x-x_0)^k , \qquad (2)$$

The following theorems that can be deduced from equations (1) and (2) are given below

THEOREM 3. If $y(x) = y_1(x) \pm y_2(x)$, then $\mathrm{Y(k) = Y_1(k) \pm Y_2(k)}$.

THEOREM 4. If $y(x) = cy_1(x)$, then $\mathrm{Y(k) = cY_1(k)}$, where c is constant.

THEOREM 5. If $y(x) = \frac{d^n y_1(x)}{dx^n}$, then $Y(k) = \frac{(k+n)!}{k!}Y_1(k+n)$.

THEOREM 6. If $y(x) = y_1(x)y_2(x)$, then $Y(k) = \sum_{k_1=0}^{k} Y_1(k_1)Y_2(k-k_1)$.

THEOREM 7. If $y(x) = x^n$, then $Y(k) = \delta(k-n)$ where

$$\delta(k-n) = \begin{cases} 1 & k = n \\ 0 & k \neq n \end{cases}$$

THEOREM 8. If $y(x) = e^{\omega x}$, then $Y(k) = \frac{\omega^k}{k!}$, where ω is constant.

THEOREM 9. If $y(x) = \sin(\omega x + \alpha)$, then $Y(k) = \frac{\omega^k}{k!}\sin\left(\frac{k\pi}{2} + \alpha\right)$, where ω and α constants .

THEOREM 10. If $y(x) = \cos(\omega x + \alpha)$, then $Y(k) = \frac{\omega^k}{k!}\cos\left(\frac{k\pi}{2} + \alpha\right)$, where ω and α constants .

4- BASIC IDEA OF THE (TDTM)

We'll study the solution of Euler's equation of second order with two independent variables.

To illustrate the basic idea of this method , we consider the general form of one-dimensional non-homogeneous partial differential equations with a variable coefficients of the form

$$a_0 t^2 u_{tt} + a_1 t u_t + a_2 x u_x + a_3 x^2 u_{xx} + a_4 u = f(x,t), \qquad \dots(3)$$

where $u = u(x,t), \; t > 1, \; x > 0$,

subject to

$$u(x,1) = g_1(x), \qquad u_t(x,1) = g_2(x) \qquad \dots(4)$$

$$u(0,t) = h_1(t), \qquad u_x(0,t) = h_2(t) \qquad \dots(5)$$

The methodology consists of applying a Temimi transforms to equations (3) – (5), and by the initial conditions (4), we get

$$(a_0(s-2)(s-1) + a_1(s-1) + a_4)v(x,s) - a_0 g_2(x) - (a_0(s-2)+a_1)g_1(x) + a_2 x \frac{dv}{dx} + a_3 x^2 \frac{d^2v}{dx^2} = \tilde{f}(x,s) \qquad \dots(6)$$

Subject to

$$v(0,s) = \overline{h_1}(s), \qquad \frac{dv}{dx}(0,s) = \overline{h_2}(s) \qquad \dots(7)$$

Which is second_ order initial value problem.

According to Differential transform method, the solution of (6),(7) can be written as :

$$v(x,s) = \sum_{k=0}^{\infty} U(k) x^k \qquad \dots(8)$$

Where $U(k)$ is the differential transform of $v(x,s)$, and $U(k)$ is a function of the parameter s . Once $v(x,s)$ is determined, we then apply the inverse Temimi transform to (8), to get $u(x,t)$.

5- Applications and Results

In this section, we demonstrate the effectiveness of the TDTM. The advantage of the proposed method is its capability of combining the two powerful methods for obtaining exact solutions for several illustrative examples .

5.1 Example

To solve the equation:

$$t u_t = u + x u_x , \qquad x \in R, t > 1 \qquad \dots(9)$$

subject to

$$u(x,1) = 3x^2 + 1$$
…(10)

and

$$u(0,t) = t\,, \qquad u_x(0,t) = 0$$
…(11)

By applying the a foresaid method subject to the initial conditions (10),we have

$$(s-2)v(x,s) - 3x^2 - 1 = x\frac{dv}{dx}$$
…(12)

Subject to

$$v(0,s) = \frac{1}{s-2}\,, \qquad \frac{dv}{dx}(0,s) = 0 \qquad \text{…(13)}$$

Now, we apply the Differential transform to equations (12)-(13) to get

$$(s-2)U(k) - 3\delta(k-2) - \delta(k) = \sum_{i=0}^{k}(i+1)U(i+1)\delta(k-i-1)\,, \quad k \geq 0$$

…(14)

And

$$U(0) = \frac{1}{s-2}\,, \qquad U(1) = 0$$
…(15)

By the above recurrence equation (14) and the initial (15), we get the following outcome

$$U(2) = \frac{3}{s-4}\,, \qquad U(k) = 0 \;:\, k \geq 3.$$
…(16)

Therefore

$$v(x,s) = U(0) + U(2)x^2$$

$$= \frac{1}{s-2} + \frac{3}{s-4}x^2$$
…(17)

Apply inverse Temimi transform to (17) yields

$$u(x,t) = t + 3t^3x^2 \qquad \text{…(18)}$$

Which is the solution for the problem (9)

5.2 Example

For solve the equation

$t^2 u_{tt} = \frac{1}{2} x^2 u_{xx}$, $x \in R, t > 1$ …(19)

Subject to $u(x,1) = x$, $u_t(x,1) = x^2$ …(20)

$u(0,t) = 0$, $u_x(0,t) = 1$ …(21)

By using (20), the Temimi transform of (19) is

$$(s-1)(s-2)v(x,s) - \frac{1}{2} x^2 \frac{d^2 v}{dx^2} = x^2 + (s-2)x \qquad \dots(22)$$

Subject to

$$v(0,s) = 0 , \quad \frac{dv}{dx}(0,s) = \frac{1}{s-1} \qquad \dots(23)$$

Apply DT to (22)-(23) yields

$$(s-1)(s-2)U(k) - \frac{1}{2}\sum_{i=0}^{k}(i+1)(i+2)U(i+2)\delta(k-i-2) = \delta(k-2) + (s-2)\delta(k-1) \ : k \geq 0, \qquad \dots(24)$$

and

$$U(0) = o , \quad U(1) = \frac{1}{s-1} \qquad \dots(25)$$

By the above recurrence equation (24) and the initial (25), we get the following outcome

$$U(2) = \frac{1}{\left(s-\frac{3}{2}\right)^2 - \frac{5}{4}} , \quad U(k) = 0 : k \geq 3 \qquad \dots(26)$$

Therefore

$$v(x,s) = U(1)x + U(2)x^2$$

$$= \frac{1}{s-1} x + \frac{1}{\left(s-\frac{3}{2}\right)^2 - \frac{5}{4}} x^2 \qquad \dots(27)$$

Apply inverse Temimi transform to (27) yields

$$u(x,t) = x + \frac{2}{\sqrt{5}} x^2 t^{\frac{1}{2}} sinh \frac{\sqrt{5}}{2} lnt \qquad \dots(28)$$

Which is the solution of the problem (19).

5.3 Example

To solve the equation

$$t^2u_{tt} + tu_t + xu_x - x^2u_{xx} - u = x^2lnt + xt\ , \qquad x \in R, t > 1 \qquad \ldots(29)$$

$$u(x,1) = 0\ , \qquad u_t(x,1) = x \qquad \ldots(30)$$

$$u(0,t) = 0\ , \qquad u_x(0,t) = t - 1 \qquad \ldots(31)$$

By using (30), the Temimi transform of (29) is

$$s(s-2)v(x,s) + x\frac{dv}{dx} - x^2\frac{d^2v}{dx^2} = \frac{x^2}{(s-1)^2} + \frac{x}{s-2} + x \qquad \ldots(32)$$

Subject to

$$v(0,s) = 0\ , \qquad \frac{dv}{dx}(0,s) = \frac{1}{s-2} - \frac{1}{s-1} \qquad \ldots(33)$$

Apply DT to (32)-(33) yields

$$s(s-2)U(k) + \sum_{i=0}^{k}(i+1)u(i+1)\delta(k-i-1) - \sum_{i=0}^{k}(i+1)(i+2)U(i+2)\delta(k-i-2) = \frac{\delta(k-2)}{(s-1)^2} + \frac{\delta(k-1)}{s-2} + \delta(k-1) \ \ : k \geq 0, \qquad \ldots(34)$$

$$U(0) = 0\ , \qquad U(1) = \frac{1}{s-1} + \frac{1}{s-2} \qquad \ldots(35)$$

By the above recurrence equation (34) and the initial (35), we get the following outcome

$$U(2) = \frac{1}{s(s-2)(s-1)^2}\ , \qquad U(k) = 0 : k \geq 3 \qquad \ldots(36)$$

Therefore

$$v(x,s) = U(1)x + U(2)x^2$$

$$= \frac{1}{s-1}x + \frac{1}{s-2}x + \frac{1}{s(s-2)(s-1)^2}x^2 \qquad \ldots(37)$$

Apply inverse Temimi transform to (37) yields

$$u(x,t) = -x + xt - \frac{1}{2}x^2t^{-1} + \frac{1}{2}x^2t - x^2lnt \qquad \ldots(38)$$

Which is the solution of (29).

5.4 Example

To solve the equation

$t^2u_{tt} - xu_x = \frac{1}{2}(coslnt + sinlnt)$, $x \in R, t > 1$ …(39)

$u(x,1) = x^2$, $u_t(x,1) = 0$
…(40)

$u(0,t) = \frac{-1}{2} sinlnt + t - \frac{1}{2}$, $u_x(0,t) = 0$
…(41)

By using (40), the Temimi transform of (39) is

$(s-1)(s-2)v(x,s) - x\frac{dv}{dx} = \frac{1}{2}x\left(\frac{s-1}{(s-1)^2+1}\right) + (s-2)x^2$
…(42)

Subject to

$v(0,s) = \frac{-1}{2}\left(\frac{1}{(s-1)^2+1}\right) + \frac{1}{s-2} + \frac{-1}{2(s-1)}$, $\frac{dv}{dx}(0,s) = 0$
…(43)

Apply DT to (42)-(43) yields

$(s-1)(s-2)U(k) + \sum_{i=0}^{k}(i+1)u(i+1)\delta(k-i-1) = \frac{1}{2}\delta(k)\left(\frac{s}{(s-1)^2+1}\right) + (s-2)\delta(k-2)$: $k \geq 0$, …(44)

$U(0) = \frac{-1}{2}\left(\frac{1}{(s-1)^2+1}\right) + \frac{1}{s-2} + \frac{-1}{2(s-1)}$, $U(1) = 0$
…(45)

By the above recurrence equation (44) and the initial (45), we get the following outcome

$U(2) = \frac{s-2}{s(s-3)}$, $U(k) = 0 : k \geq 3$ …(46)

Therefore

$v(x,s) = U(0) + U(2)x^2$

$= \frac{-1}{2}\left(\frac{1}{(s-1)^2+1}\right) + \frac{1}{s-2} + \frac{-1}{2(s-1)} + \frac{s-2}{s(s-3)}x^2$
…(47)

Apply inverse Temimi transform to (47) yields

$u(x,t) = -\frac{1}{2} sinlnt + t - \frac{1}{2} + \frac{2}{3}x^2t^{-1} + \frac{1}{3}x^2t^2$
…(48)

Which is the solution of (39).

Conclusion

A combined form of the Temimi transform method with the (DTM) is effectively used to handle the above examples of non- homogeneous linear (PDES) the solution have been obtained with just the two first terms of the (TDTM) solution , which indicate that the proposed method (TDTM) need much less computational work compared with the slandered (DTM).

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تركيب تحويل التميمي والتحويل التفاضلي لحل المعادلات التفاضلية الجزئية الخطية غير المتجانسة

بواسطة

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المستخلص

هدفنا في هذا البحث هو جمع (تركيب)صيغة تحويل التميمي مع التحويل التفاضلي لحل المعادلات التفاضلية الجزئية الخطية غير المتجانسة ذات المعاملات المتغيرة .