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Studying of Transitions symmetry shapes for $^{148-156}$Nd Isotopes using interacting boson model

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Abstract:

The symmetry states structure of even -even $^{148-156}$Nd isotopes has been studied using the interacting boson model (IBM-1) .The energy levels , the electromagnetic transitions probability B(E2) and potential energy surfaces are analyzed which reveal the detailed nature of nuclei.

In this chain ^{148}Nd nucleus have a transition characteristic between vibrational limit SU(5)& γ- unstable limits O(6) .

Different behavior shown by ^{150} Nd nuclei where the quadrupole – quadrupole interactions between bosons and the pairing forces were dominated while the studied structure band of nuclei $^{152-156}$Nd looks as distorted nuclei.

The predicted theoretical calculations were compared with the experimental data in respective figures and tables ,it was seen that the predicted results are in a good agreement with the experimental data.

In the framework of IBM calculations (46) new energy levels were determined for even -even $^{148-156}$Nd isotopes .This investigation increases the theoretical Knowledge of all isotopes with respect to energy levels ,reduced transition probabilities and potential energy surfaces.

دراسة شكل التماثلات الانتقالـــــية لنوى $^{148-156}$Nd باستخدام أنموذج البوزونات المتفاعلة

الخلاصة:

تمت دراسة تركيب الحالات التماثلية لنظائر $^{148-156}$Nd باستخدام نموذج البوزونات المتفاعلة الأول. حللت مستويات الطاقة ،احتمالية الانتقالات الكهرومغناطيسية B(E2) ،وسطوح تساوي الجهد التي تظهر الطبيعة المفصلة للنوى .

في هذه السلسلة تمتلك نواة ^{148}Nd صفات انتقالية بين التحديين الاهتزازي SU(5) و تحديد كاما غير المستقرة O(6). أن سلوكا مغايرا تظهره نواة ^{150} Nd حيث يكون تفاعلات عزم باعي القطب –عزم رباعي القطب بين البوزونات وقوى الازدواج هي المهيمنة في حين ان تركيب حزم الطاقة المدروسة لنوى $^{152-156}$Nd تبدو كنوى مشوهه.

الحسابات النظرية المتوقعة قورنت مع البيانات العملية بجداول ورسومات خاصة ويبدو أن النتائج المتوقعة متوافقة جيدا مع البيانات العملية.
في نطاق حسابات IBM (46) مستوي طاقة جديد قد حدد لنظائر $^{148-156}$Nd الزوجية - الزوجية. هذا البحث قد زاد المعرفة النظرية بجميع هذه النظائر بالنسبة لـ مستويات الطاقة واحتمالية الانتقالات المختزلة وجهد طاقة السطح.

Introduction:

In the interacting boson model ,collective excitations of nuclei are described by bosons. An appropriate formalism to describe the situation is provided by second quantization . One thus introduces boson creation (and annihilation)operators of multi polarity l and z- component m .A boson model is specified by the number of bosons operators that are introduced .In the interacting boson model -1 it is assumed that low –lying collective states of nuclei can described in terms of a monopole bosons with angular momentum and parity $J^P = 0^+$,called s and a quadrupole boson with $J^P = 2^+$ called d[1-6].

There are two basic concepts on which the IBM is based. One is that low-lying collective states in even-even nuclei can be described by only the valence nucleons, which form interacting fermion pairs. The other idea is that the fermion pairs couple to form bosons, carrying angular momentum (J). The energies (ε_s and ε_d), and the interactions of the s and d bosons, predict the low-lying excitations in the nucleus. There is 1 available magnetic substate for the s boson, determined by (2J + 1), and 5 available magnetic substates for the d boson, forming a 6-dimensional space described by the group structure[7]. The quadrupole collectivity is a prominent aspect in the nuclear structure for both stable and exotic nuclei [8,9].

The use of boson degree of freedom to describe the quantum dynamics of many fermion systems is a vast subject .The interacting boson model of Arima and Iachello has been successfully applied to a wide range of nuclear collective phenomena . The essential idea is that the low energy collective degrees of freedom in nuclei can be described by proton and neutron bosons with spins of 0 and 2. These collective building blocks interact. Different choices of L=0 (s-boson) and L=2 (d-boson) energies and interaction strengths give rise to different types of collective spectra. The IBM is a phenomenological model ,that is to say its parameters are determined by fitting to the excitation spectra of nuclei .The interpretation of the boson as proton pairs and neutrons pairs is only manifested in the means by which N_π and N_ν are chosen for a given nucleus .There is extensive literature that undertakes to interpret the bosons of the model microscopically [1-6].

Many of studies have been done in the last few years ago on $^{148-156}$Nd , In 2000 Al-Banna,et. al. applied X(5) Model on ^{150}Nd [10]. In 2003 Abdullah et. al. and Singh et. al ,analyzed nuclear band spectra in ^{152}Nd and ^{144,146,148,150}Nd respectively in 2003

[11,12]. In (2010) the low lying energy levels for ^{150}Nd have been calculated using Density Functional Theory by Meng et.al. [13].

Interacting Boson Model (IBM):

Hamiltonian operator

The Hamiltonian operator according to IBM- 1 describes the system of s (L= 0) and d (L= 2) boson can be written either [1-6,14]

$\hat{H} = \varepsilon\, \hat{n}_d + a_0 (\hat{P}^\dagger \cdot \hat{P}) + a_1(\hat{L}\cdot\hat{L}) + a_2(\hat{Q}\cdot\hat{Q}) + a_3(\hat{T}_3\cdot\hat{T}_3) + a_4(\hat{T}_4\cdot\hat{T}_4)$……(1)

where ε, a_0, a_1, a_2, a_3, a_4 parameters used in the IBM – 1.

$\varepsilon = (\varepsilon_d - \varepsilon_s)$ ………………………………...(2)

ε – Boson energy , ε_d – energy of d – boson , ε_s – energy of s – boson

$\hat{n}_d = [\hat{d}^\dagger \cdot \tilde{\hat{d}}]$ the boson number type of (d – boson)……..… (3)

$\hat{P} = \frac{1}{2}(\tilde{\hat{d}} \cdot \tilde{\hat{d}}) - \frac{1}{2}(\hat{S}\cdot\hat{S})$ the pairing bosons operator…………… (4)

$\hat{L} = \sqrt{10}\,(\hat{d}^\dagger \times \tilde{\hat{d}})$ angular momentum operator………..… (5)

$\hat{Q} = [\hat{d}^\dagger \times \tilde{\hat{s}}]^2 + [\hat{s}^\dagger \times \tilde{\hat{d}}]^2 + \mathrm{CHI}[\hat{d}^\dagger \times \tilde{\hat{d}}]^2$ Quadrupole momentum operator... (6)

CHI = $-\sqrt{\frac{7}{2}}$ for rotational dynamical symmetry and CHI = Zero for vibrational and γ — soft dynamical symmetry.

$(\hat{T}_3) = [\hat{d}^\dagger \times \tilde{\hat{d}}]^3$ Octupole operator……………………..… (7)

$(\hat{T}_4) = [\hat{d}^\dagger \times \tilde{\hat{d}}]^4$ Hexadecapole operator………………… (8)

The creation ($\hat{d}^\dagger$, $\hat{s}^\dagger$) and annihilation ($\hat{d}, \hat{s}$) operators can be used in the commutation relations form as follows[15-18]

$$\left.\begin{array}{l} [\hat{s}, \hat{s}^\dagger] = 1,\ [\hat{s}, \hat{s}] = [\hat{s}^\dagger, \hat{s}^\dagger] = 0; \\ [\hat{d}_\mu, \hat{d}_{\mu'}^\dagger] = \delta_{\mu\mu'};\ [\hat{d}_{\mu'}, \hat{d}_{\mu'}] = [\hat{d}_{\mu'}^\dagger, \hat{d}_{\mu'}^\dagger] = 0; \\ [\hat{s}, \hat{d}_\mu^\dagger] = [\hat{s}, \hat{d}_\mu] = [\hat{s}^\dagger, \hat{d}_\mu^\dagger] = [\hat{s}^\dagger, \hat{d}_\mu] = 0 \end{array}\right\} \quad \text{………………..(9)}$$

The operator $\hat{d}$ is defined by[1-6]

$\tilde{\hat{d}}_m = (-1)^m\, \hat{d}_{-m};\ \tilde{\hat{s}} = \hat{s}$ ………………..… (10)

Or [7,9]

$$\hat{H} = \epsilon_s(\hat{S}^\dagger \cdot \tilde{\hat{S}}) + \epsilon_d(\hat{d}^\dagger \cdot \tilde{\hat{d}}) + \sum_{L=0,2,4} \frac{1}{2}\sqrt{2L+1}\, C_L \left[\left[\hat{d}^\dagger \otimes \hat{d}^\dagger\right]^L \otimes \left[\hat{d} \otimes \tilde{\hat{d}}\right]^L\right]^O$$

$$+\frac{1}{\sqrt{2}}V_2\left[\left[\hat{d}^{\dagger}\otimes\hat{d}^{\dagger}\right]^2\oplus\left[\hat{\tilde{d}}\otimes\hat{\tilde{S}}\right]^2+\left[\hat{d}^{\dagger}\otimes\hat{S}^{\dagger}\right]^2\otimes\left[\hat{d}\otimes\hat{\tilde{d}}\right]^2\right]^O$$

$$+\frac{1}{2}V_O\left[\left[\hat{d}^{\dagger}\otimes\hat{d}^{\dagger}\right]^O\otimes\left[\hat{\tilde{S}}\otimes\hat{\tilde{S}}\right]^O+\left[\hat{\tilde{S}}^{\dagger}\otimes\hat{\tilde{S}}^{\dagger}\right]^O\otimes\left[\hat{\tilde{d}}\otimes\hat{\tilde{d}}\right]^O\right]^O+U_2\left[\left[\hat{d}^{\dagger}\otimes\hat{S}^{\dagger}\right]^2\otimes\left[\hat{\tilde{d}}\otimes\hat{\tilde{S}}\right]^2\right]^O$$

$$+\frac{1}{2}U_O\left[\left[\hat{S}^{\dagger}\otimes\hat{S}^{\dagger}\right]^O\otimes\left[\hat{\tilde{S}}\otimes\hat{\tilde{S}}\right]^O\right]^O \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

Where

$\in_L (L=0,2), C_L(L=0,2,4), V_L(L=0,2), U_L(L=0,2)$

represents the boson energies and interactions.

Transition Operators

The electromagnetic transition operators can be written as[1-6, 19-21]

$$\left.\begin{aligned}
&\hat{T}_0^{(E0)} = \gamma_0 + \alpha_0\,[\hat{s}^{\dagger}\times\hat{s}]_0^{(0)} + \beta_0\,[\hat{d}^{\dagger}\times\hat{\tilde{d}}]_0^{(0)},\\
&\hat{T}_\mu^{(M1)} = \beta_1\,[\hat{d}^{\dagger}\times\tilde{d}]_\mu^{(1)},\\
&\hat{T}_\mu^{(E2)} = \alpha_2[\hat{d}^{\dagger}\times\hat{\tilde{s}} + \hat{s}^{\dagger}\times\hat{\tilde{d}}]_\mu^{(2)} + \beta_2[\hat{d}^{\dagger}\times\tilde{d}]_\mu^{(2)},\\
&\hat{T}_\mu^{(M3)} = \beta_3\,[\hat{d}^{\dagger}\times\tilde{d}]_\mu^{(3)},\\
&\hat{T}_\mu^{(E4)} = \beta_4\,[\hat{d}^{\dagger}\times\tilde{d}]_\mu^{(4)}
\end{aligned}\right\}\ldots\ldots\ldots\ldots(12)$$

where

γ_0 is the electromagnetic transition parameter

α_L (L = 0, 2)

and β_L (L= 0,1,2 ,3,4)

Are parameters specifying the various terms in the corresponding operators.

Electromagnetic transitions and moments

The most general form of electromagnetic transition operator in IBM-1 is given by electric quadrupole $\hat{T}^{(E2)}$ operator and magnetic dipole $\hat{T}^{(M1)}$ operator as follows[8,18]

Electric quadrupole operator $\hat{T}^{(E2)}$

The electric quadrupole $\hat{T}^{(E2)}$ operator can be taken from equation (2-11) as[6]

$$\hat{T}_\mu^{(E2)} = \alpha_2[\hat{d}^{\dagger}\times\hat{\tilde{s}} + \hat{s}^{\dagger}\times\hat{\tilde{d}}]_\mu^{(2)} + \beta_2[\hat{d}^{\dagger}\times\tilde{d}]_\mu^{(2)} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

where

α_2 is the effective charge of s and d boson

β_2 is the effective charge of d- boson

The selection rules of the electric quadrupole operator of the three dynamical symmetries (Chain I, II, and III) are as follows [1-6, 19-21]

Chain I $\Delta n_d = 0, \pm 1$, $\Delta\upsilon = \pm 1$, $\Delta n_\Delta = 0$, $|\Delta I| \leq 2$ SU(5)

Chain II $\Delta\lambda = 0$, $\Delta\mu = 0$ SU(3)

Chain III $\Delta\sigma = 0$, $\Delta\tau = \pm 1$ O(6)

The electric quadrupole transition rates are governed by B(E2) values, these are defined as[8]

$$B(E2; I_i \rightarrow I_f) = \frac{1}{2I+1} \left| \langle I_f \| \hat{T}^{(E2)} \| I_i \rangle \right|^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

where

I_i is the initial angular momentum ,I_f is the final angular momentum

The electric quadrupole moments Q_I of the three dynamical symmetry SU(5),SU(3) and O(6), where they are used in the present work can be written as follows[1-6]

$$Q_I = \langle I, M_I = I \left| \sqrt{\left[\frac{16\pi}{5}\right]}\, \hat{T}^{(E2)} \right| I, M_I = I \rangle \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

$$Q_I = \sqrt{\left[\frac{16\pi}{5}\right]} \begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix} \langle I_f \| \hat{T}^{(E2)} \| I_i \rangle \text{ in SU(5) Limit} \ldots\ldots (16)$$

$$Q_I = -\alpha_2 \sqrt{\left[\frac{16\pi}{40}\right]} I / 2I + 3(4N+3) \qquad \text{in SU(3) Limit} \ldots\ldots (17)$$

$$Q_I = 0 \qquad \text{in O(6) Limit} \ldots\ldots (18)$$

and the (Branching Ratios) of the reduced transition probabilities for electric quadrupole transition of the dynamical symmetry SU(5), SU(3), and O(6) obeys the following relations [18-21]

$$R = B(E2; 4_1^+ \rightarrow 2_1^+) / B(E2; 2_1^+ \rightarrow 0_1^+) \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

$$R' = B(E2; 2_2^+ \rightarrow 2_1^+) / B(E2; 2_1^+ \rightarrow 0_1^+) \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

$$R'' = B(E2; 0_2^+ \rightarrow 2_1^+) / (E2; 2_1^+ \rightarrow 0_1^+) \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

In addition, they can be written in terms of the total boson number (11):-

$$R = R' = R'' = \frac{2(N-1)}{N} \xrightarrow[N\rightarrow\infty]{} 2 \qquad \text{in SU(5) Limit} \ldots\ldots\ldots (22)$$

$$\left.\begin{aligned} R &= \frac{10}{7}\frac{(N-1)(2N+5)}{N(2N+3)} \xrightarrow[N\rightarrow\infty]{} \frac{10}{7}; \\ R' &= R'' = 0 \end{aligned}\right\} \text{in SU(3) Limit} \ldots\ldots (23)$$

$$\left.\begin{aligned} R = R' &= \frac{10}{7}\frac{(N-1)(N+5)}{N(N+4)} \xrightarrow[N\rightarrow\infty]{} \frac{10}{7}; \\ R'' &= 0 \end{aligned}\right\} \text{in O(6) Limit} \ldots\ldots (24)$$

where N – is the total boson number.

Potential Energy Surface(P.E.S.)

The general formula for the potential energy surface as a function of geometrical variables β and γ is given by [7,9]

$$V(\beta,\gamma) = \frac{N(\epsilon_S + \epsilon_d\ \beta^2)}{1+\beta^2} + \frac{N(N-1)}{(1+\beta^2)^2}(\alpha_1\beta^4 + \alpha_2\beta^3\cos 3\gamma + \alpha_3\beta^2 + \alpha_4) \quad \ldots(25)$$

where

N= is the total boson number

$\hat{\beta}2$ = is the quadrupole deformation parameter operator from 0 → 2.4

$\hat{\gamma}$ = is the distortion parameter operator or (asymmetry angle) for $0^o \rightarrow 60^o$ as shown in Fig. (1) .The variables (α_1, α_2, α_3, α_4) are related to the parameters C_L, V_L, and U_L which is given in equation (11).

The relation ships between the variables (α's) and these parameters have been expressed [20-22] :-

$$\alpha_1 = \frac{C_0}{10} + \frac{C_2}{7} + \frac{9}{35} * C_4 \quad \ldots\ldots (26)$$

$$\alpha_2 = -SQRT(\frac{8}{35}) * V_2 \quad \ldots\ldots (27)$$

$$\alpha_3 = (V_0 + U_2)/SQRT(5) \quad \ldots\ldots (28)$$

$$\alpha_4 = U_0 \quad \ldots\ldots (29)$$

One must take into account that the asymmetry angle occurs only in the term cos 3γ. Thus, the energy surfaces has minima only at $\gamma = 0^o$ and 60^o .The energy expressions in their limits, can display the essential dependence and β and γ, which have been given as [7,9] :-

For large N, β_{min} =0, √2 and 1 for SU(5),SU(3) and O(6) respectively.

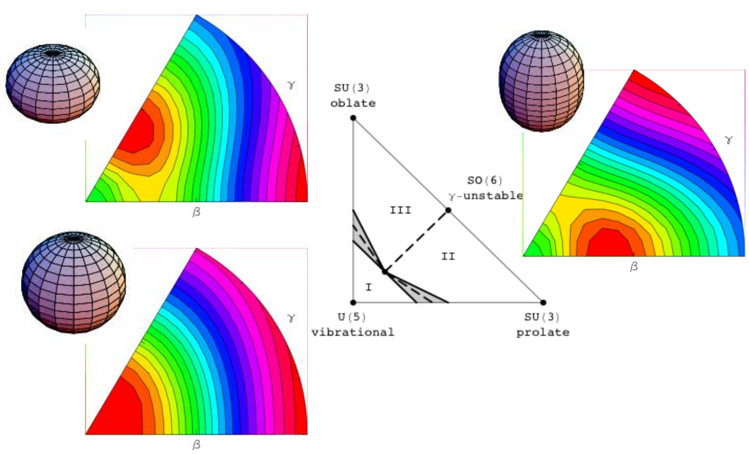


Fig.(1) : Surface phase transitions diagram of IBM [22].

Calculations and results:

Calculations of energy levels for even-even $^{148-156}$Nd isotopes were performed with the whole Hamiltonian (eq.1) using IBM-1 computer code . For $^{148-156}$Nd nuclei (Z=60) have (8-12) bosons formed (5 proton particle) bosons and (3-7) neutron particle bosons where N= (88-96).

The parameters of equation (1) were calculated from the experimental schemes of these nuclei [23-36] and the analytical solutions for the three dynamical systems (see reference [4]). These parameters were tabulated in table (1) . The calculated and experimental energy levels and the parameters value were exhibit in figure(2)&(3).

The calculations of B(E2) values were performed using computer code "IBMT". The parameters in E2 operator eq. (13) were determined by fitting the experimental $B(E2;2_1^+\rightarrow 0_1^+)$ data [23-36], and the parameters were listed in table(2),where

$\beta_2 = \frac{-0.7}{5}\alpha_2, -\sqrt{7/2}\,\alpha_2$ *and* $=0$ $E2SD = \alpha_2, E2DD = \sqrt{5}\beta_2$ and

in SU(5), SU(3) and O(6) respectively[4-7]. With addition to Comparison between present values of B(E2) (in unit e^2b^2) for even-even $^{148-156}$Nd isotopes (Theo.) and experimental ones (Exp.) [23-36], the quadrupole moment of 2_1^+ state listed in last line.

The converter coefficient between (e^2b^2) and (W.u) is $B(E2)w.u = \frac{B(E2)e^2b^2}{5.943\times10^{-6}A^{4/3}e^2b^2}$.

Table (1): The parameters of the Hamiltonian equation used for the description of the $^{148-156}$Nd isotopes.

Parameters / Isotope	N_b	ε	a_0	a_1	a_2	a_3	a_4
		In (MeV)					
^{148}Nd	8	0.38	0.015	0.01	0.0	0.007	0.0007
^{150}Nd	9	0.0	0.05	0.015	-0.0125	0.0001	0.0001
^{152}Nd	10	0.0	0.0	0.005	-0.019	0.0001	0.0001
^{154}Nd	11	0.0	0.0	0.0065	-0.0141	0.0001	0.0001
^{156}Nd	12	0.0	0.0	0.0052	-0.0159	0.0001	0.0001

Table (2): Comparison between present values of B(E2) (in unit e^2b^2) for even-even $^{148-156}$Nd isotopes (Theo.) and experimental ones (Exp.) [19-29].The quadrupole moment of 2_1^+ state listed in last line.

Transitions	E2DD	E2SD	$2_1^+\rightarrow 0_1^+$		$2_2^+\rightarrow 0_1^+$		$2_2^+\rightarrow 2_1^+$		$4_1^+\rightarrow 2_1^+$		$Q2_1^+$	
Isotope	In unit (e^2b^2)		Th.	Exp.	Th.	Exp.	Th.	Exp.	Th.	Exp.	Th.	Exp.
^{148}Nd	0.105	-0.001	0.28	0.277	0.0024	0.0025	0.0042	-	0.39	0.43	-1.46	-1.41
^{150}Nd	0.14	-0.03	0.58	0.54	0.0031	0.002	0.0054	-	0.81	0.82	-2	-2
^{152}Nd	0.12	-0.15	0.764	0.768	0.0	-	0.0	-	1.07	1.04	-2.3	-
^{154}Nd	0.089	-0.13	0.464	0.464	0.0	-	0	-	0.65	-	-1.82	-
^{156}Nd	0.086	-0.12	0.421	-	0.0	-	0.0	-	0.59	-	-1.73	-

The parameters of the energy surface were calculated by transforming the parameters of Hamiltonian of equation (1) by several equations (see reference [4]), and they are found to be as in table (3) to draw the energy functional E(N; β,γ) as a function of β and the contour plots in the □-β plane fig.(4).

Table (3): The parameters of the Hamiltonian equation used in the programs IBMP code for potential energy surface

Parameters	N_b	ε	a_0	a_1	a_2	a_3	a_4
Isotope		In (MeV)					
^{148}Nd	8	0.38	0.015	0.01	0.0	0.007	0.0007
^{150}Nd	9	0.0	0.05	0.015	-0.0125	0.0001	0.0001
^{152}Nd	10	0.0	0.0	0.005	-0.019	0.0001	0.0001
^{154}Nd	11	0.0	0.0	0.0065	-0.0141	0.0001	0.0001
^{156}Nd	12	0.0	0.0	0.0052	-0.0159	0.0001	0.0001

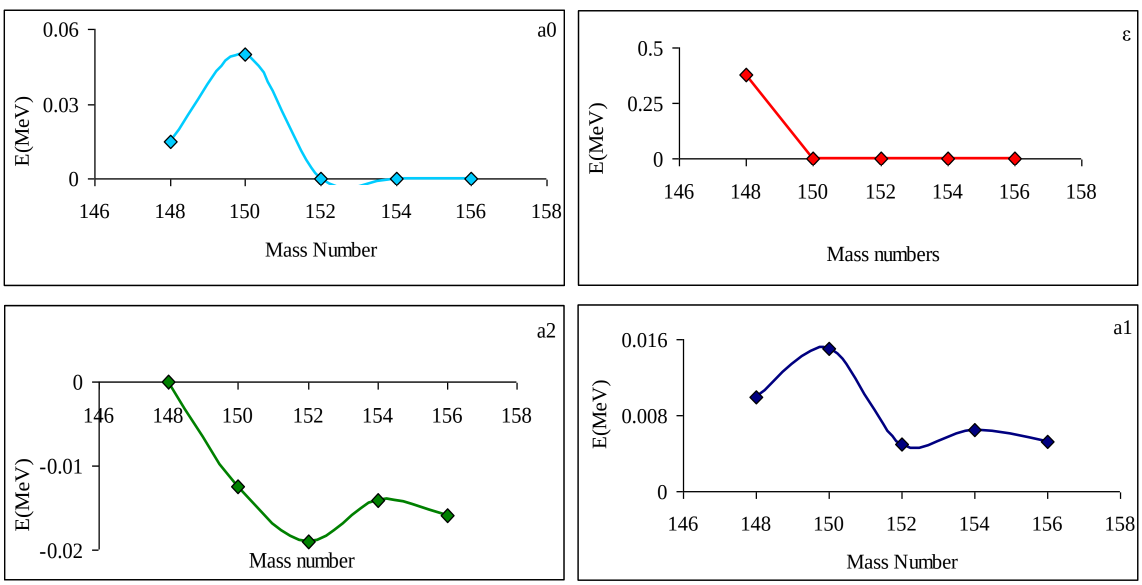
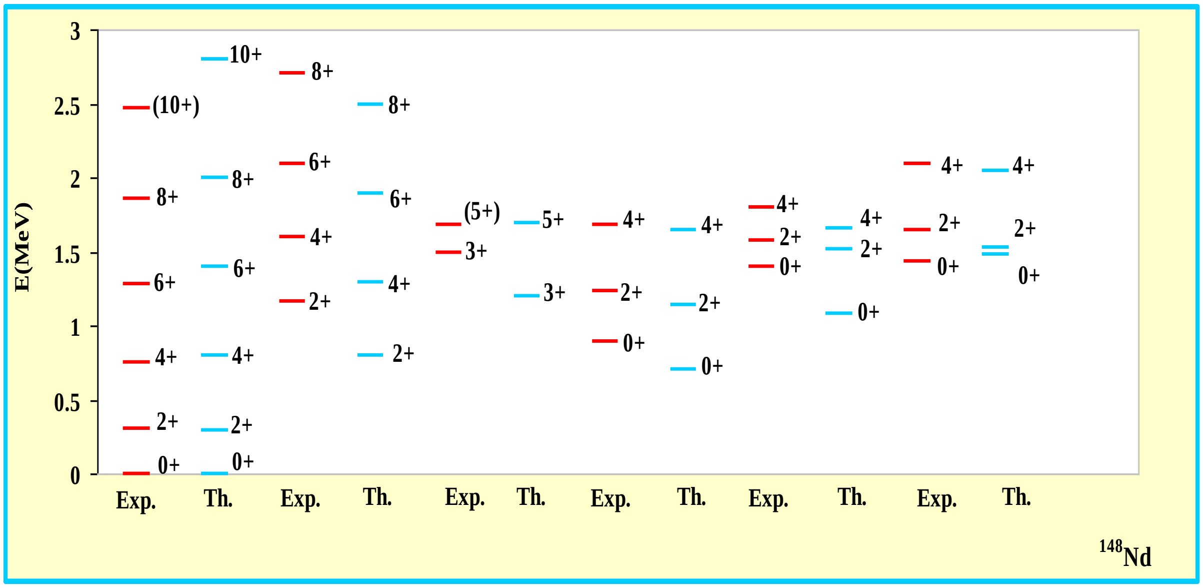
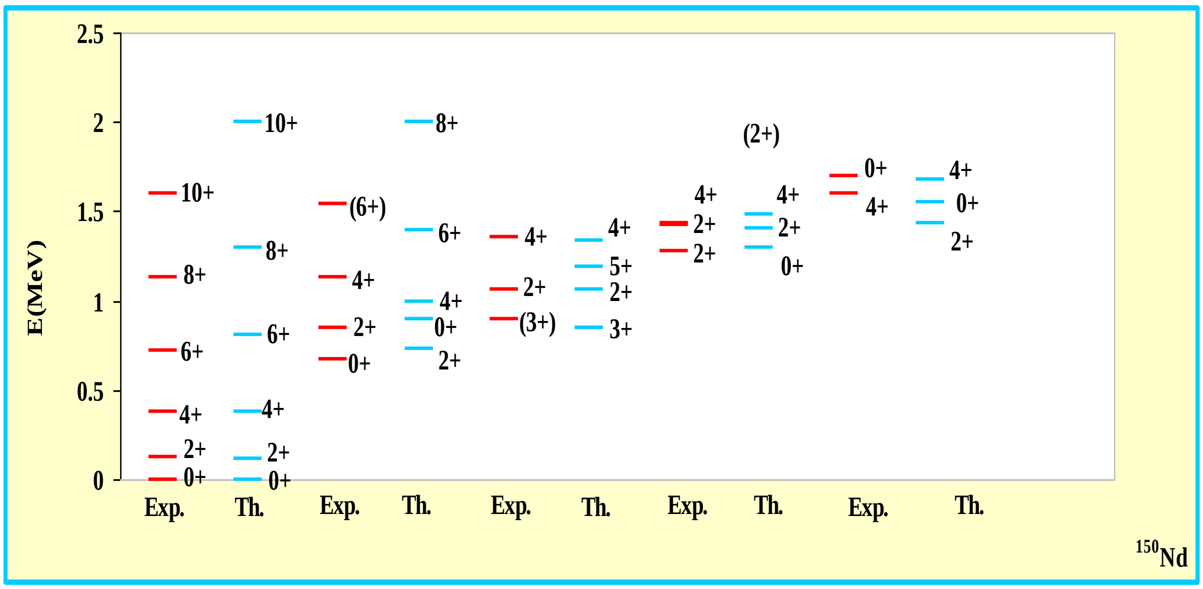
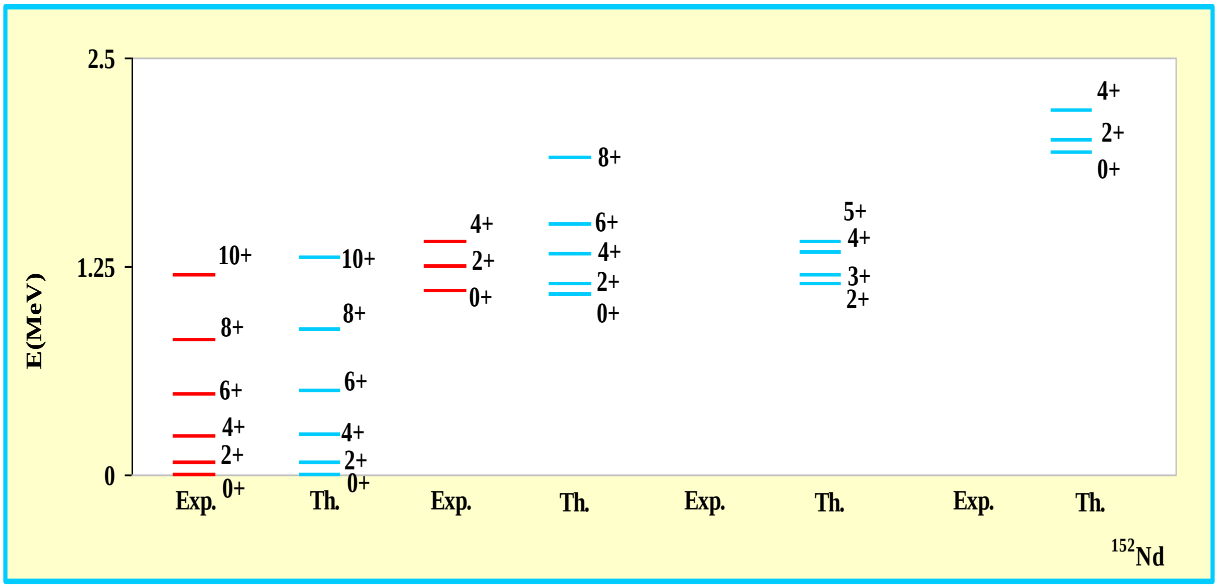
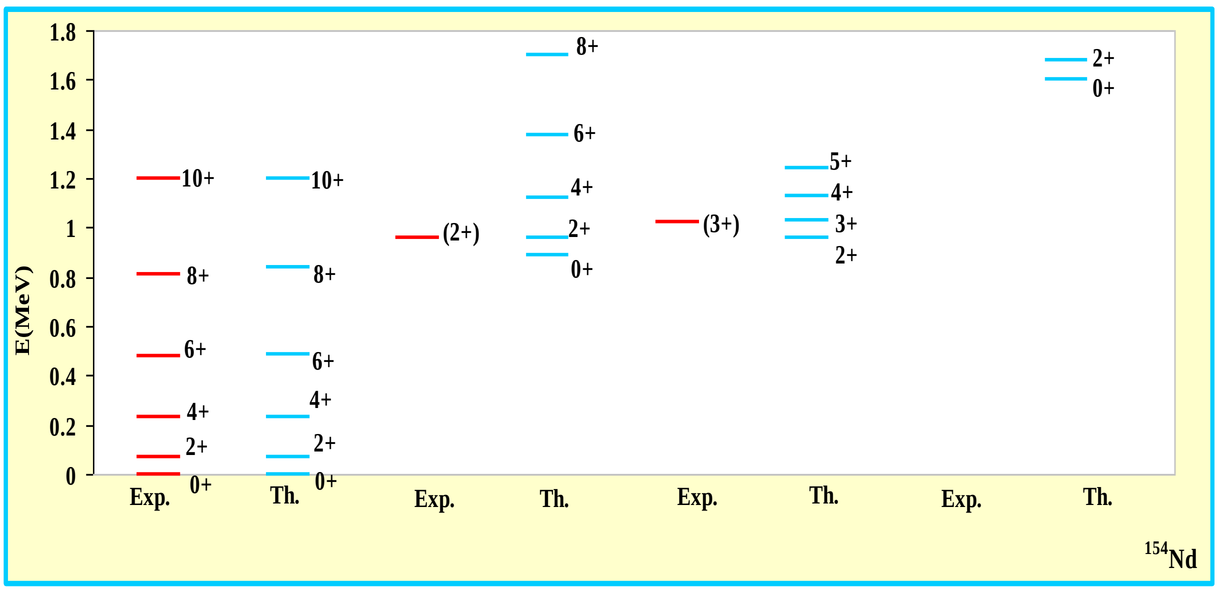


Fig.(2): The values of the parameters (ε, a_0, a_1 and a_2) were calculated from the experimental schemes of $^{148-156}$Nd isotopes.


E(MeV)
3
2.5
2
1.5
1
0.5
0
Exp.
Th.
(10+)
8+
6+
4+
2+
0+
10+
148Nd


E(MeV)
2.5
2
1.5
1
0.5
0
Exp.
Th.
10+
8+
6+
4+
2+
0+
(6+)
(3+)
(2+)
150Nd


2.5
1.25
0
E(MeV)
10+
8+
6+
4+
2+
0+
4+
2+
0+
8+
6+
5+
3+
Exp.
Th.
152Nd


1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
E(MeV)
10+
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3+
Exp.
Th.
154Nd

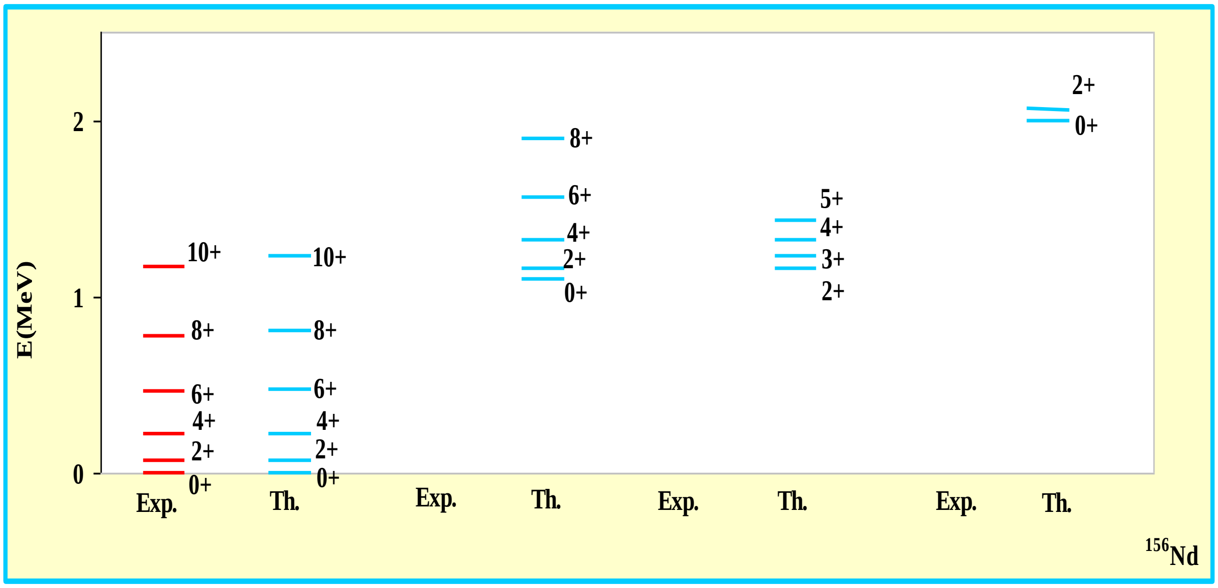
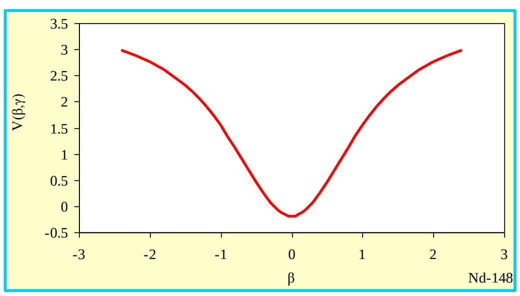
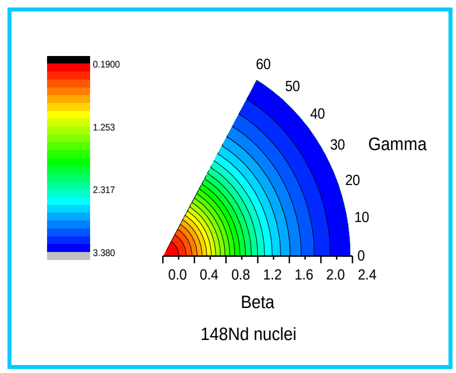
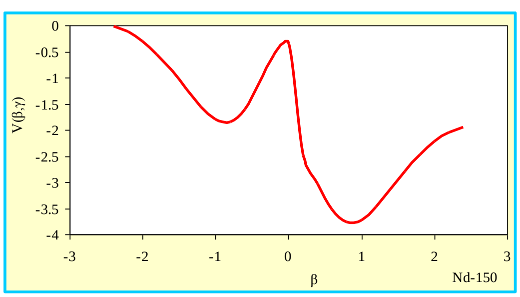
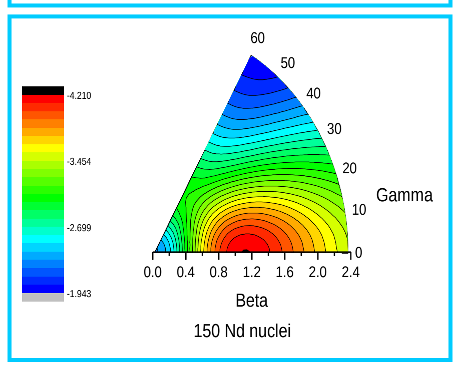


Fig. (3): A comparison between theoretical values of energy levels and the corresponding experimental one for $^{148-156}$Nd .









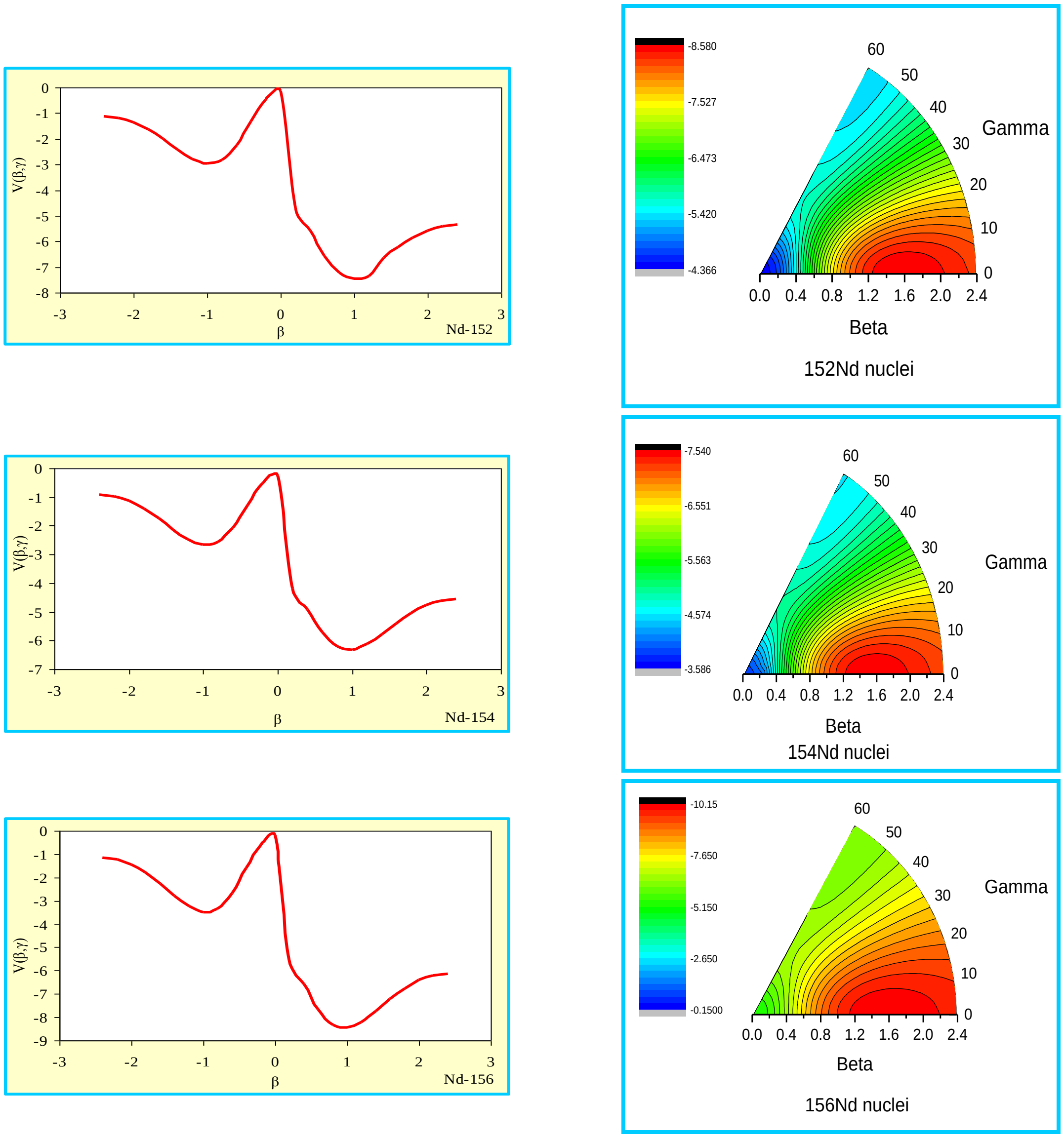


Fig.(4):The energy functional E(N; β,γ) as a function of β and the corresponding β-γ counter plot for $^{148-156}$Nd isotopes.

Discussion and Conclusion:

The energy ratio $E(J_i^\pi)/E(2_1^+)$ for the $J_i^\pi = 4_1^+, 6_1^+$ and 8_1^+ levels for the doubly even Neodymium isotopes were shown on figure (5). ^{148}Nd nuclei was leave SU(5) limit to O(6) limit because of the experimental and calculated ratio values E^+_4/E^+_2, E^+_6/E^+_2 & E^+_8/E^+_2 were as (2.49,4.24 and 6.15) .Different behavior shown by on ^{150} Nd nuclei where the ratio values were as follows(2.92, 5.53 and 8.67) which means an intermediate ratio between O(6) limit and SU(3) limit where the quadrupole – quadrupole interaction between bosons and the pairing forces were dominated . These forces especially influence the particles in the unfilled states .The pairing force keeps the nuclei in spherical symmetry .The quadrupole charge distribution causes what is known as the quadrupole force ,this force take the nuclei to the deformed state ,the relation between the pairing and the quadrupole forces determines the form of the nuclei.

The studied structure band of nuclei $^{152-156}Nd$ looks as distorted nuclei where the rates of practical and theoretical energies ($4_1^+ / 2_1^+$) were (3.26 and 3.29, and 3.3) and (6.67, 6.8 and 6.85) for the ratio ($6_1^+ / 2_1^+$) and (11.1, 11.44 and 11.58) for the ratio ($8_1^+ / 2_1^+$) respectively, which are virtually identical to the SU(3) rates specifically, as shown in Figure (5) .The behavior of these ratios of the energy leave the SU(5) limit to O(6) limit toward the SU(3) limit. Deformed nuclei can exhibit rotational spectra ,which depend on the nuclear equilibrium shape.

The gradation in the $^{148-156}Nd$ nuclei characteristics can be interpreted if we look at the neutron distribution in the nuclear shells, the $^{148-152}Nd$ nuclei occupy $1h_{9/2}$ while $^{154,156}Nd$ occupy the $2f_{7/2}$ sub levels respectively .

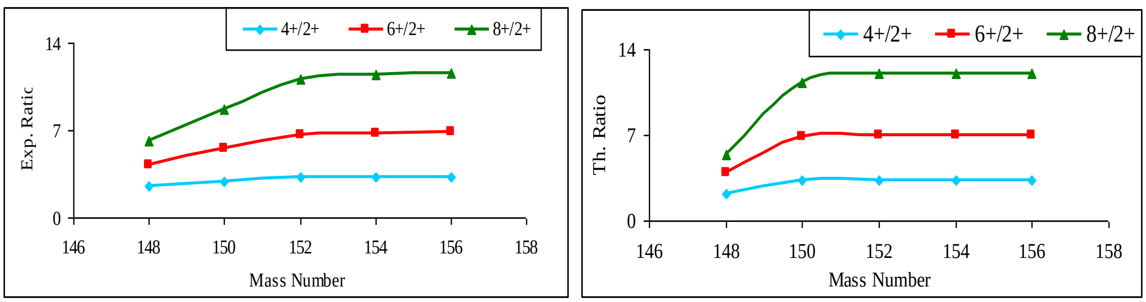


Fig.(5):Calculated and Experimental ratios ($4^+/2^+$),($6^+/2^+$) and ($8^+/2^+$) for $^{148-156}Nd$ isotopes.

In the present work, the researcher has been applied the geometrical model of the IBM–1 for even–even nuclei, since the IBM–1 geometrical model of nuclear collective motion provided as with an alternative description of nuclear collective excitations, which are more sensitive than the phenomenological model.

The IBM–1 analysis of the counter plots of the potential energy function V(β, γ), calculated by using the parameters (α's) that deduced from (IBMP. For) program, as shown in table (3). Figure (4) elucidates the potential energy surfaces as a function of deformed parameters (β, γ), the contour lines are in good agreement with the typical plots as shown in figure (1). The axially symmetric ($\gamma = 0^o$, $\gamma = 30^o$, $\gamma = 60^o$) plots of the potential function, calculated in the present work, for $^{148-156}Nd$ isotopes show that:

1. The axially symmetric for the isotope ^{148}Nd of the dynamical symmetry U(5)–O(6), figure (4) shows the behaviors of the potential energy surfaces with a minimum of (0.198 MeV) on the prolate shape at $\beta = 0$, $\gamma = 0^o$, and the same value for oblate shape at, $\gamma=60^o$. Also shows good agreement with the typical axially symmetric of U(5)–O(6) limits.

2. From the axially symmetric for the isotope ^{140}Nd of the dynamical symmetry O(6)-SU(3) , as shown in figure (4), the behavior of the potential energy surfaces of (3.76 MeV) on the prolate side at β=0.8, γ=0^o and for ablate shape at β = 0.8, γ = 60^o which is (1.84 MeV). This is agree with the behavior of the U(5) limit compared with the typical triaxial symmetric.
3. The axially symmetric for the isotope $^{152-156}$Nd of the dynamical symmetry SU(3), figure (4) shows the behavior of the potential energy surfaces with a minimum of (7.4 MeV,6.3MeV and 8.45 MeV) respectively on the prolate shape, at at β = 1.2, γ= 0^o. This is agreeing with the typical axially symmetric of the SU (3) limit.

In the framework of IBM calculations (46) new energy levels were determined for even -even $^{146-156}$Nd isotopes as (0^+_3:1.32 MeV , 4^+_5 :2.42MeV, 6^+_2 :2.1MeV and 5^+_1 :2.04MeV for ^{146}Nd , (0^+_3 :1.07MeV, 0^+_4 :1.48MeV , 10^+_1 :2.47MeV and 5^+_1 :1.74MeV) for ^{148}Nd, (0^+_3 :1.28MeV ,0^+_4 :1.7MeV , 2^+_4 :1.42MeV , 3^+_1: 0.93 MeV, 6^+_2 :1.54MeV, 2^+_5 :1.44 and 5^+_1 :1.19MeV) for ^{150}Nd, (6^+_2 :1.59MeV , 8^+_2 :1.95MeV , 2^+_3 :1.15MeV, 3^+_1 :1.22MeV , 4^+_3 :1.32MeV ,5^+_1 :1.44MeV, 0^+_3 :1.93MeV , 2^+_4 :2.01MeV and4^+_4 :2.18) for ^{152}Nd, (0^+_2 :0.89 MeV , 4^+_2 :1.12MeV , 6^+_2 :1.38MeV , 8^+_2 :1.73MeV , 2^+_3: 0.9 MeV,4^+_3 :1.12MeV , 5^+_1 :1.24MeV , 0^+_3 :1.61MeV and 2^+_4:1.68MeV) for ^{154}Nd and (0^+_2 :1.07 MeV , 2^+_2 :1.16MeV , 4^+_2 :1.3MeV , 6^+_2 :1.56MeV , 8^+_2: 1.9 MeV,2^+_3 :1.165 MeV , 3^+_1 :1.23 MeV , 4^+_3 :1.321MeV , 5^+_1:1.43MeV, 0^+_3 :2 MeV and 2^+_4 :2.07 MeV,) for ^{154}Nd. see fig.(3).

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